# 5 Abstract coordinate systems

## 5.1 Introduction

An abstract coordinate system is a means of identifying positions in position-space by coordinate $n$-tuples. An abstract coordinate system is completely defined in terms of the mathematical structure of position-space. In this International Standard the term “coordinate system”, if not otherwise qualified, is defined to mean “abstract coordinate system.” Each coordinate system has a coordinate system type (see 5.4). Other coordinate system related concepts defined in this clause include coordinate-component surfaces and coordinate-component curves (see 5.5), linearity and other properties (see 5.6), and localization (see 5.7). Map projections and augmented map projections are defined and treated as special cases of the general abstract coordinate system concept (see 5.8). Standardized abstract coordinate systems are specified in 5.9.

In Clause 6 a temporal coordinate system is defined as a means of identifying events in the time continuum by coordinate 1-tuples using an abstract coordinate system of coordinate system type 1D. In Clause 8 a spatial coordinate system is defined as an abstract coordinate system suitably combined with a normal embedding (see Clause 7) as a means of identifying points in object-space by coordinate $n$-tuples.

## 5.2 Preliminaries

This International Standard takes a functional approach to the construction of coordinate systems. Annex A provides a concise summary of mathematical concepts and specifies the notational conventions used in this International Standard. In particular, Annex A defines the terms interior, one-to-one, smooth, smooth surface, smooth curve, orientation-preserving, and connected. The concept of $\mathbf{R}^n$ as a vector space, the point-set topology of $\mathbf{R}^n$, and the theory of real-valued functions on $\mathbf{R}^n$ are all assumed. Algebraic and analytic geometry, including the concepts of point, line, and plane, are also assumed. Together with such common concepts, a newly introduced concept replete will be used. A set $D$ is replete if all points in $D$ belong to the closure of the interior of $D$ (see Annex A). A replete set is a generalization of an open set that allows the inclusion of boundary points. Boundary points are important in the definitions of certain coordinate systems.

## 5.3 Abstract CS

An *abstract Coordinate System* (CS) is a means of identifying a set of positions in an abstract Euclidean space that shall be comprised of:

a) a CS domain,

b) a generating function, and

c) a CS range,

where:

a) The *CS domain* shall be a connected replete domain in the Euclidean space of $n$-tuples ($1 \leq n \leq m$), called the *coordinate-space.*

b) The *generating function* shall be a one-to-one, smooth, orientation-preserving function from the CS domain onto the CS range.

c) The *CS range* shall be a set of positions in a Euclidean space of dimension $m$ ($n \le m \le 3$), called the *position-space.* When $n = 2$ and $m = 3$, the CS range shall be a subset of a smooth surface[5]. When $n = 1$ and $m = 2$ or 3, the CS range shall be a subset of an implicitly specified smooth curve[6].

An element of the CS domain shall be called a *coordinate*[7]. The $k^{th}$-component of a coordinate $n$-tuple ($1 \le k \le n$) may be called the $k^{th}$ *coordinate-component*. *Coordinate-component*[8] is the collective term for any $k^{th}$ coordinate-component.

An element of the CS range shall be called a *position.* The *coordinate of a position* $\boldsymbol{p}$ shall be the unique coordinate whose generating function value is $\boldsymbol{p}$.

The generating function may be parameterized. The generating function parameters (if any) shall be called the *CS parameters.*

The inverse of the generating function shall be called the *inverse generating function*. The inverse generating function is one-to-one and is smooth and orientation-preserving in the interior of its domain, except at points in the image of the CS domain boundary points where it may be discontinuous. A CS may equivalently be defined by specifying the inverse generating function when the CS domain is an open set.

NOTE 1 The generating function of a CS is often specified by an algebraic and/or trigonometric description of a geometric relationship (see 5.3 Example). There are also CSs that do not have geometric derivations. The Mercator map projection (see Table 5.18) is specified to satisfy a functional requirement of conformality (see 5.8.3.2) rather than by a geometric construction.

EXAMPLE Polar CS: Considering the polar geometry depicted in Figure 5.1, define a generating function $\boldsymbol{F}$ as:

$$\boldsymbol{F}(\rho,\ \theta) = (x,\ y)$$

where:

$$x = \rho\cos(\theta), \text{ and } \ y = \rho\sin(\theta).$$

The CS domain of $\boldsymbol{F}$ in coordinate-space is $\{(\rho,\theta) \text{ in } \mathbf{R}^2 \mid 0 < \rho,\ 0 \le \theta < 2\pi\} \cup \{(0,0)\}$.
The CS range of $\boldsymbol{F}$ in position-space is $\mathbf{R}^2$.

This generating function is illustrated in Figure 5.2. The grey boxes with lighter grey edges in this figure represent the fact that the range in position-space extends indefinitely, and that the domain in coordinate-space extends indefinitely along the $\rho$-axis. The dotted grey edges indicate an open boundary. This CS range, CS domain, and generating function define an abstract CS representing polar coordinates as defined in mathematics [EDM, “Coordinates”]. The normative definition of the polar CS may be found in Table 5.33.

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[5] The generating function properties and the implicit function theorem together imply that for each point in the interior of the CS domain, there is an open neighbourhood of the point whose image under the generating function lies in a smooth surface. This requirement specifies that there exists one smooth surface for all of the points in the CS domain. This requirement is specified to exclude mathematically pathological cases.

[6] The generating function properties and the implicit function theorem together imply that for each point in the interior of the CS domain, there is an open neighbourhood of the point whose image under the generating function lies in a smooth curve. This requirement specifies that there exists one implicitly-defined smooth curve for all the points in the CS domain. This requirement is specified to exclude mathematically pathological cases.

[7] The ISO 19111 term for this concept is “coordinate tuple”.

[8] The ISO 19111 term for this concept is “coordinate”.

 

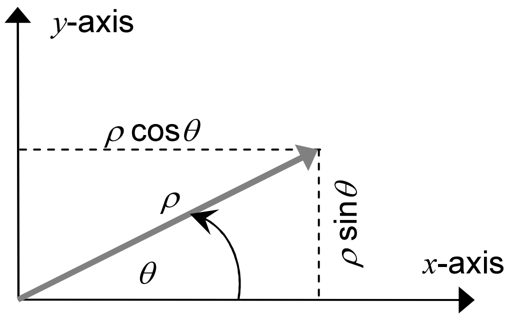


**Figure 5.1 — Polar CS geometry**

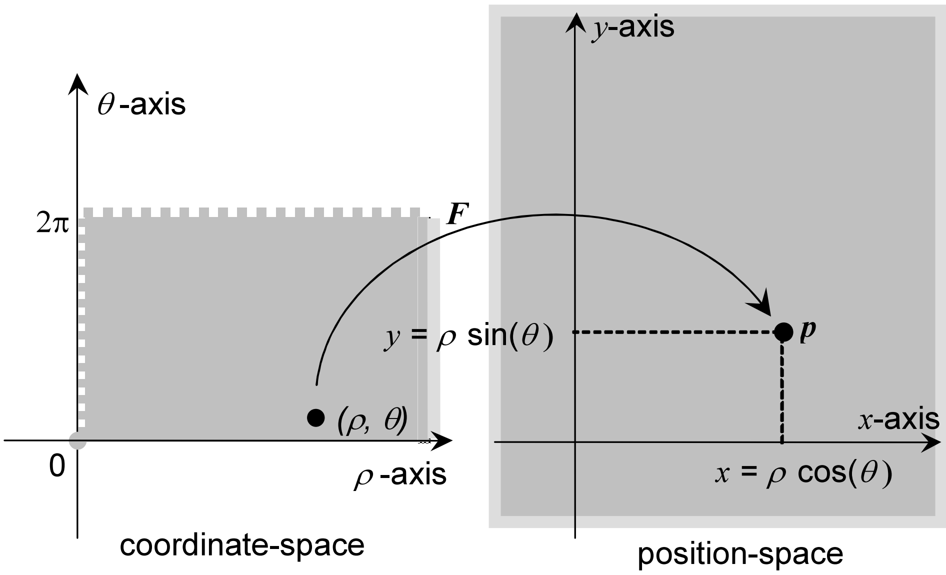


**Figure 5.2 — The polar CS generating function**

NOTE 2 In the special case where 1) the CS domain and CS range are both $\mathbf{R}^n$ and 2) the function is the identity function, this approach to defining coordinate systems reduces to the usual definition of the Euclidean coordinate system on $\mathbf{R}^n$ where each point is identified by an $n$-tuple of real numbers [EDM] (see Table 5.8, Table 5.29 and Table 5.35).

NOTE 3 The CS generating function has an inverse because it is one-to-one, but the inverse may be discontinuous at points in the image of CS domain boundary points. This is the case for the positive $x$-axis in the example above.

## 5.4 CS types

The coordinate-space and position-space dimensions characterize an abstract CS by *CS type* as defined in Table 5.1.

**Table 5.1 — CS types**

| CS type | Dimension of coordinate-space | Dimension of position-space |
|---|---|---|
| 3D | 3 | 3 |

| CS type | Dimension of coordinate-space | Dimension of position-space |
|---|---|---|
| surface | 2 | 3 |
| curve[9] | 1 | 3 |
| 2D | 2 | 2 |
| plane curve[9] | 1 | 2 |
| 1D | 1 | 1 |

A CS of CS type 3D may be called a 3D CS, a CS of CS type surface may be called a surface CS, and a CS of CS type 2D may be called a 2D CS.

## 5.5 Coordinate surfaces, induced surface CSs, and coordinate curves

### 5.5.1 Introduction

The generating function of a 3D CS is a function of the three coordinate-components of a coordinate 3-tuple. If one of the coordinate-components is held fixed (to a constant value), then the generating function thus restricted to two variables may be viewed as a surface CS generating function (with a surface CS range). If two of the three coordinate-components are held fixed, the generating function restricted to one variable may be viewed as a curve CS generating function (with curve CS range). These observations motivate the definitions of coordinate-component surfaces and curves. The coordinate-component surface and coordinate-component curve concepts are required to specify induced CS relationships, for the definition of special coordinate curves parallel and meridian, and the definition of CS handedness (see also 10.5).

### 5.5.2 Coordinate-component surfaces and induced surface CSs

If $\boldsymbol{F}$ is the generating function of a 3D CS, and $\boldsymbol{u} = (u_0, v_0, w_0)$ is in the interior of the CS domain $D$, then three surface CS generating functions at $\boldsymbol{u}$ are defined by:

$$\boldsymbol{S}_1(v,w) = \boldsymbol{F}(u_0,v,w),$$

$$\boldsymbol{S}_2(u,w) = \boldsymbol{F}(u,v_0,w), \text{ and}$$

$$\boldsymbol{S}_3(u,v) = \boldsymbol{F}(u,v,w_0).$$

The CS domain for $\boldsymbol{S}_1$ is the connected component of $\{(v,w) \text{ in } \mathbf{R}^2 \mid (u_0,v,w) \text{ in } D\}$ which contains $(v_0, w_0)$.

The CS domain for $\boldsymbol{S}_2$ is the connected component of $\{(u,w) \text{ in } \mathbf{R}^2 \mid (u,v_0,w) \text{ in } D\}$ which contains $(u_0, w_0)$.

The CS domain for $\boldsymbol{S}_3$ is the connected component of $\{(u,v) \text{ in } \mathbf{R}^2 \mid (u,v,w_0) \text{ in } D\}$ which contains $(u_0, v_0)$.

Each of these surface CSs shall be called, respectively, the 1st, 2nd, and 3rd *surface CS induced by* $\boldsymbol{F}$ at $\boldsymbol{u}$.

The CS ranges of these surface CSs are, respectively, the 1st, 2nd, and 3rd *coordinate-component surface at* $\boldsymbol{u}$.

EXAMPLE 1 Coordinate-component surface: The geodetic 3D CS with generating function $\boldsymbol{F}(\lambda,\varphi,h) = (x,y,z)$ is specified in Table 5.14 with CS parameters $a$ and $b$. The 3rd coordinate-component surface at $\boldsymbol{u} = (\lambda_0,\varphi_0,0)$ is the surface of the oblate ellipsoid with major semi-axis $a$ and minor semi-axis $b$.

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[9] The ISO 19111 concept of a linear reference system is a specialization of the curve CS and plane curve CS concepts.

 

EXAMPLE 2 Induced surface CS: The surface geodetic CS is specified in Table 5.24. Its CS domain, CS range and generating function are identical to the 3rd surface CS induced by the geodetic 3D generating function at $\boldsymbol{u} = (0,0,0)$. If $h$ is replaced by 0 in the formulae for the generating and inverse generating functions of the geodetic 3D CS, they reduce to the surface geodetic formulae.

### 5.5.3 Coordinate-component curves

Coordinate-component curves are defined for CSs of CS type 3D, CS type surface, and CS type 2D.

The CS type 3D case:

If $\boldsymbol{F}$ is the generating function of a CS of CS type 3D, $D$ is the CS domain, and $\boldsymbol{u} = (u_0, v_0, w_0)$ is in the interior of $D$, then the 1st, 2nd, and 3rd *coordinate-component curves at* $\boldsymbol{u}$ are parametrically specified, respectively, by the following smooth functions:

$$\boldsymbol{C}_1(u) = \boldsymbol{F}(u, v_0, w_0),$$

$$\boldsymbol{C}_2(v) = \boldsymbol{F}(u_0, v, w_0), \text{ and}$$

$$\boldsymbol{C}_3(w) = \boldsymbol{F}(u_0, v_0, w).$$

The domain for $\boldsymbol{C}_1$ is the connected component of $\{u \text{ in } \mathbf{R} \mid (u, v_0, w_0) \text{ in } D\}$ which contains $u_0$.

The domain for $\boldsymbol{C}_2$ is the connected component of $\{v \text{ in } \mathbf{R} \mid (u_0, v, w_0) \text{ in } D\}$ which contains $v_0$.

The domain for $\boldsymbol{C}_3$ is the connected component of $\{w \text{ in } \mathbf{R} \mid (u_0, v_0, w) \text{ in } D\}$ which contains $w_0$.

NOTE The intersection of two coordinate surfaces at $u$ is (the locus of) a coordinate-component curve: $C_1 = S_2 \cap S_3$, $C_2 = S_1 \cap S_3$, $C_3 = S_1 \cap S_2$.

The CS type surface and CS type 2D cases:

If $\boldsymbol{F}$ is the generating function of a CS of CS type surface or CS type 2D, $D$ is the CS domain, and $\boldsymbol{u} = (u_0, v_0)$ is in the interior of $D$, then the 1st and 2nd *coordinate-component curves at* $\boldsymbol{u}$ are parametrically specified, respectively, by the following smooth functions:

$$\boldsymbol{C}_1(u) = \boldsymbol{F}(u, v_0), \text{ and}$$

$$\boldsymbol{C}_2(v) = \boldsymbol{F}(u_0, v).$$

The domain for $\boldsymbol{C}_1$ is the connected component of $\{u \text{ in } \mathbf{R} \mid (u, v_0) \text{ in } D\}$ which contains $u_0$.

The domain for $\boldsymbol{C}_2$ is the connected component of $\{v \text{ in } \mathbf{R} \mid (u_0, v) \text{ in } D\}$ which contains $v_0$.

EXAMPLE If $\boldsymbol{u} = (\rho_0, \theta_0)$ is in the interior of the CS domain of the polar CS generating function $\boldsymbol{F}$ of the 5.3 Example, then the first coordinate-component curve is $\boldsymbol{C}_1(\theta) = \boldsymbol{F}(\rho_0, \theta) = (\rho_0 \cos\theta,\ \rho_0 \sin\theta)$, and the 2nd coordinate-component curve is $\boldsymbol{C}_2(\theta) = \boldsymbol{F}(\rho, \theta_0) = (\rho\cos\theta_0,\ \rho\sin\theta_0)$.

If $\boldsymbol{F}$ is the generating function for the geodetic 3D CS or the surface geodetic CS, and $\boldsymbol{u} = (\lambda_0, \varphi_0, 0)$ in the 3D case or $\boldsymbol{u} = (\lambda_0, \varphi_0)$ in the surface case, then (see Figure 5.3):

a) the 1st coordinate-component curve at $\boldsymbol{u}$ shall be called the *parallel* at $\boldsymbol{u}$, and

 

b) the 2nd coordinate-component curve at $\boldsymbol{u}$ shall be called the *meridian*[10] at $\boldsymbol{u}$.

The meridian at $\boldsymbol{u} = (0,0,0)$ or $(0,0)$ shall be called the *prime meridian*[11].

The parallel at $\boldsymbol{u} = (0,0,0)$ or $(0,0)$ shall be called the *equator*.

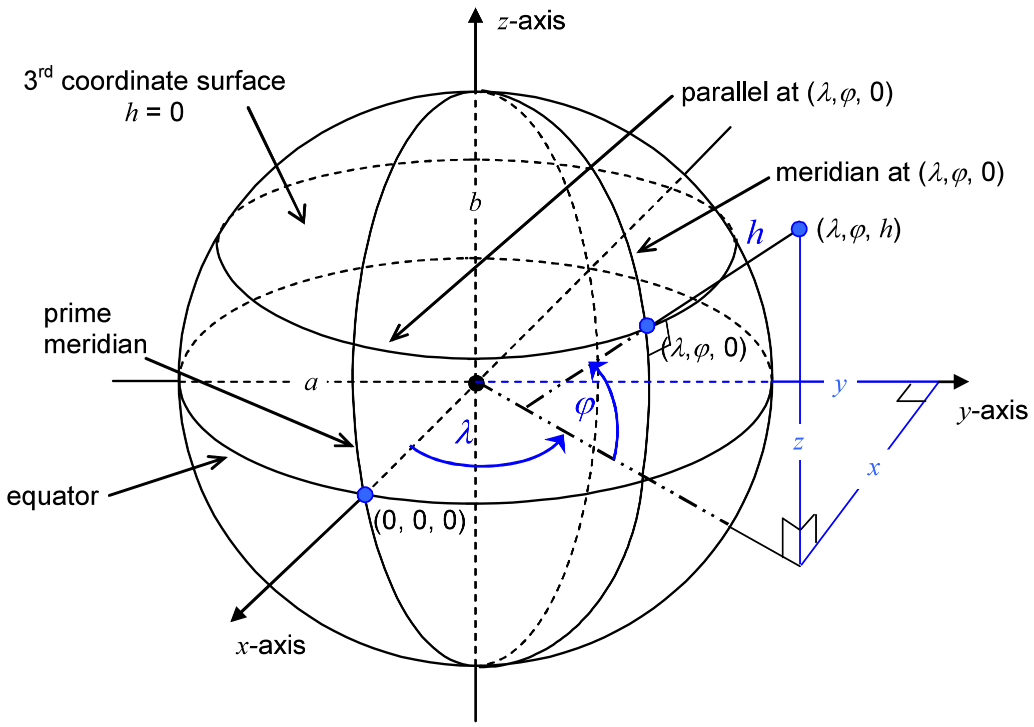


**Figure 5.3 — Geodetic 3D CS geometry, and coordinate-component surface and curves**

## 5.6 CS properties

### 5.6.1 Linearity

A CS with generating function $\boldsymbol{F}$ is a *linear CS* if $\boldsymbol{F}$ is an affine function. The CS domain of a linear coordinate system is all of the coordinate-space $\mathbf{R}^n$.

A *curvilinear CS* is a non-linear CS.

EXAMPLE The polar CS of 5.3 EXAMPLE is a curvilinear CS of CS type 2D.

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[10] ISO 19111 defines the term "meridian" as the intersection between an ellipsoid and a plane containing the semi-minor axis of the ellipsoid.

[11] ISO 19111 defines the term "prime meridian" as the meridian from which the longitudes of other meridians are quantified. In Clause 7, most, but not all, oblate ellipsoid Earth object reference models associate the Greenwich meridian with the prime meridian (see 7.4.5).

 

### 5.6.2 Orthogonality

A CS of CS type 3D, CS type surface, or CS type 2D is *orthogonal* if the angle between any two coordinate-component curves at $\boldsymbol{u}$ is a right angle when $\boldsymbol{u}$ is any coordinate in the interior of the CS domain of the generating function.

EXAMPLE The polar CS of 5.3 EXAMPLE is a orthogonal CS of CS type 2D.

### 5.6.3 Linear CS properties: Cartesian, and orthonormal

In a linear CS, the $k^{\text{th}}$ coordinate-component curve is a line. The $k^{\text{th}}$ coordinate-component curve at the origin **0** of a linear CS is the *$k^{th}$-axis*.

In a linear CS, if the angles between coordinate-component curves at the origin **0** are (pair-wise) right angles, then that is the case at all points. In particular, a linear CS is orthogonal[12] if the axes are orthogonal.

In some publications a Cartesian CS is defined the same way as an orthogonal linear CS[13]. This International Standard, however, defines this concept differently. A linear CS with generating function $\boldsymbol{F}$ is a *Cartesian CS* if $\left\| \boldsymbol{F}(e_i) - \boldsymbol{F}(\mathbf{0}) \right\| = 1, \quad i = 1,\ldots,n$ (*i.e.,* the axis unit points are all one unit distant from the origin $\boldsymbol{F}(\mathbf{0})$).

An *orthonormal CS* is a linear CS that is both orthogonal and Cartesian.

A CS of CS type 3D with generating function $\boldsymbol{F}$ is *orientation-preserving* if the Jacobian determinant of $\boldsymbol{F}$ is positive.

EXAMPLE The Lococentric Euclidean 3D CS specified in Table 5.9 is an orientation-preserving orthonormal CS.

### 5.6.4 CS right-handedness and coordinate-component ordering

Given a CS of CS type 3D and a coordinate $\boldsymbol{c} = (u_0, v_0, w_0)$ in the interior of the CS domain, the coordinate-component curves at $\boldsymbol{p}$ determine an ordered set of three tangent vectors:

$$\boldsymbol{t}_1 = \left( \frac{d\boldsymbol{C}_1}{du} \right)_{u=u_0},$$

$$\boldsymbol{t}_2 = \left( \frac{d\boldsymbol{C}_2}{dv} \right)_{v=v_0}, \text{ and}$$

$$\boldsymbol{t}_3 = \left( \frac{d\boldsymbol{C}_3}{dw} \right)_{w=w_0}.$$

An orthogonal CS of CS type 3D is a *right-handed CS* if for some coordinate $\boldsymbol{c}$ in the interior of the CS domain, the ordered set of tangent vectors $\boldsymbol{t}_1$, $\boldsymbol{t}_2$, and $\boldsymbol{t}_3$ form a right-handed coordinate system as defined in ISO 80000-2. The right-handed CS property is determined, in part, by the order of the coordinate-components in the coordinate 3-tuple. The order of the coordinate-components in the specification of an orthogonal CS of CS type 3D shall be restricted to an ordering that ensures a right-handed CS. This restriction is required for uniform treatment of directions in an SRF (see 10.5).

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[12] Some publications use "rectangular" to denote an orthogonal linear CS, and "oblique" to denote a non-orthogonal linear CS.

[13] ISO 19111 defines "Cartesian coordinate system" as a coordinate system that gives the position of points relative to $n$ mutually-perpendicular axes.

 

The coordinate-component ordering in the specification of a surface CS that is induced on a coordinate-component surface of a 3D CS, shall use the coordinate-component order of the inducing 3D CS.

EXAMPLE 1 The geodetic 3D CS (Table 5.14) coordinate-component ordering $(\lambda, \varphi, h)$ ensures that the CS is right-handed. A similar ordering for the planetodetic 3D CS (Table 5.15) is not right-handed because the tangent to planetodetic longitude points opposite to the direction of the tangent to geodetic longitude. Instead, the coordinate-component ordering $(\varphi, \lambda, h)$ is specified to satisfy the right-handed CS requirement.

EXAMPLE 2 The surface planetodetic geodetic CS (Table 5.25) coordinate-component ordering $(\varphi, \lambda)$ is determined by the coordinate-component ordering $(\varphi, \lambda, h)$ of the planetodetic 3D CS (Table 5.15).

## 5.7 CS localization

In some applications of a CS in the context of a spatial reference frame, it is necessary to consider a modified version of the CS that has been translated to a local origin and/or has been rotated (see "Lococentric" spatial reference frame variants in Clause 8). To treat these modifications in a uniform manner, the generating function of a CS that has been translated to a local origin and/or has been rotated is related to the generating function of the original CS by means of a localization operator. This uniform method, defined below, of specifying the variant CS by composing the original CS generating function with a localization operator shall be called *CS localization*.

Three parameterized operators, called *localization operators,* that operate on or between position-spaces are defined in Table 5.2. The inverses of these operators are defined in Table 5.3.

**Table 5.2 — Localization operators**

| Localization operator | Domain | Range | Localization parameters | Operator definition |
|---|---|---|---|---|
| $L_{3D}$ | $\mathbf{R}^3$ | $\mathbf{R}^3$ | $\boldsymbol{q}$, $\boldsymbol{r}$, $\boldsymbol{s}$, in $\mathbf{R}^3$<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal | $L_{3D}(x, y, z) = \boldsymbol{q} + x\boldsymbol{r} + y\boldsymbol{s} + z\boldsymbol{t}$,<br>where $\boldsymbol{t} = \boldsymbol{r} \times \boldsymbol{s}$. |
| $L_{\text{Surface}}$ | $\mathbf{R}^2$ | $\mathbf{R}^3$ | $\boldsymbol{q}$, $\boldsymbol{r}$, $\boldsymbol{s}$, in $\mathbf{R}^3$<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal | $L_{\text{Surface}}(x, y) = \boldsymbol{q} + x\boldsymbol{r} + y\boldsymbol{s}$ |
| $L_{2D}$ | $\mathbf{R}^2$ | $\mathbf{R}^2$ | $\boldsymbol{q}$, $\boldsymbol{r}$, $\boldsymbol{s}$, in $\mathbf{R}^2$<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal | $L_{2D}(x, y) = \boldsymbol{q} + x\boldsymbol{r} + y\boldsymbol{s}$ |

**Table 5.3 — Localization inverse operators**

| Localization operator | Inverse operator definition |
|---|---|
| $L_{3D}$ | $L_{3D}^{-1}(p) = ((p - q) \bullet r)e_1 + ((p - q) \bullet s)e_2 + ((p - q) \bullet t)e_3$ |
| $L_{\text{Surface}}$ | $L_{\text{Surface}}^{-1}(p) = ((p - q) \bullet r)e_1 + ((p - q) \bullet s)e_2$ |
| $L_{2D}$ | $L_{2D}^{-1}(p) = ((p - q) \bullet r)e_1 + ((p - q) \bullet s)e_2$ |

There are several forms of CS localization depending on CS type and localization operator. A 3D or surface CS with generating function $\boldsymbol{F}$ is localized by composing $\boldsymbol{F}$ with the $L_{3D}$ localization operator. The localized CS is of the same CS type (CS type 3D or CS type surface, respectively). Its generating function is $\boldsymbol{F}_{\mathrm{L}} \equiv \boldsymbol{L}_{\mathrm{3D}} \circ \boldsymbol{F}$ and has the same CS domain as $\boldsymbol{F}$.

There are two localization operators for a 2D CS. One uses localization parameters in $\mathbf{R}^3$ and produces a surface CS. The other uses localization parameters in $\mathbf{R}^2$ and produces a 2D CS.

a) A 2D CS with generating function $\boldsymbol{F}$ is localized by composing $\boldsymbol{F}$ with the $\boldsymbol{L}_{\mathrm{Surface}}$ localization operator. The localized CS is a surface CS. Its generating function is $\boldsymbol{F}_{\mathrm{L}} \equiv \boldsymbol{L}_{\mathrm{Surface}} \circ \boldsymbol{F}$ and has the same CS domain as $\boldsymbol{F}$.

b) A 2D CS with generating function $\boldsymbol{F}$ is localized by composing $\boldsymbol{F}$ with the $\boldsymbol{L}_{\boldsymbol{2D}}$ localization operator. The localized CS is a 2D CS. Its generating function is $\boldsymbol{F}_{\mathrm{L}} \equiv \boldsymbol{L}_{\mathrm{2D}} \circ \boldsymbol{F}$ and has the same CS domain as $\boldsymbol{F}$.

The localization operator parameter $\boldsymbol{q}$ shall be called the *lococentre*. A localized CS may be called a *lococentric* CS.

NOTE CS localization preserves the following CS properties: linear/curvilinear, orthogonal, Cartesian, and orthonormal.

The relationship between a CS type and its localized version(s) is summarized in Table 5.4.

**Table 5.4 — Localized CS type relationships**

| CS type | Localization operator | Lococentric CS type |
|---|---|---|
| 3D | $\boldsymbol{L}_{\mathrm{3D}}$ | 3D |
| Surface | $\boldsymbol{L}_{\mathrm{3D}}$ | Surface |
| 2D | $\boldsymbol{L}_{\mathrm{Surface}}$ | Surface |
| 2D | $\boldsymbol{L}_{\mathrm{2D}}$ | 2D |

## 5.8 Map projection coordinate systems

### 5.8.1 Map projections

Map projections are 2D models of a 3D curved surface. In this International Standard, map projections are limited to the surface of an oblate ellipsoid. A *map projection* (MP) is comprised of

a) an MP domain in the surface of an oblate ellipsoid,

b) a generating projection, and

c) an MP range in 2D coordinate-space,

where:

a) the MP domain is a connected subset of the surface of the oblate ellipsoid,

b) the MP range is a connected replete set, and

c) the *generating projection* is one-to-one from the MP domain in the oblate ellipsoid onto its MP range and its inverse function is smooth and orientation-preserving in the MP range interior.

NOTE 1 This definition may be generalized to any ellipsoid including tri-axial ellipsoids, but this International Standard only addresses map projections for oblate ellipsoids.

NOTE 2 The domain of a map projection is always a proper subset of the oblate ellipsoid surface. In particular, the domain of the Mercator map projection (see Table 5.18) omits the pole points.

The generating projection $\boldsymbol{P}$ is specified in terms of surface geodetic CS coordinates (see Table 5.24). The component functions $P_1$ and $P_2$ of the generating projection $\boldsymbol{P}$ shall be called the *mapping equations*:

$$\boldsymbol{P}(\lambda, \varphi) = (u, v)$$

where:

$u = P_1(\lambda, \varphi)$, and

$v = P_2(\lambda, \varphi)$.

The MP range coordinate-components $u$ and $v$ shall be called *easting* and *northing*, respectively. The positive direction of the $u$-axis (the easting axis) shall be called *map-east*. The positive direction of the $v$-axis (the northing axis) shall be called *map-north*.

The inverse mapping equations are the component functions $Q_1$ and $Q_2$ of the inverse generating projection $\boldsymbol{Q} = \boldsymbol{P}^{-1}$:

$$\lambda = Q_1(u, v)$$

$$\varphi = Q_2(u, v)$$

### 5.8.2 Map projection as a surface CS

If the inverse generating projection of a map projection $\boldsymbol{Q}$ is composed with the surface geodetic CS generating function $\boldsymbol{G}_{\mathrm{GD}}$, the resulting function $\boldsymbol{G}_{\mathrm{MP}} = \boldsymbol{G}_{\mathrm{GD}} \circ \boldsymbol{Q}$ is the generating function of a surface CS (see Figure 5.4). The CS domain is the MP range. In this International Standard, a *map projection CS* shall be a surface CS for which the generating function is implicitly specified in terms of the mapping equations of a map projection.

In some cases, the surface geodetic coordinates with coordinate-component $\varphi = \pm\pi/2$ are not in the MP domain of $\boldsymbol{P}$ nor are they in the range of $\boldsymbol{Q}$. However, if the composite function $\boldsymbol{G}_{\mathrm{MP}}^{-1} = \boldsymbol{P} \circ \boldsymbol{G}_{\mathrm{GD}}^{-1}$ is continuous at the pole points $(0, 0, \pm b)$, then $\boldsymbol{G}_{\mathrm{MP}}$ and $\boldsymbol{G}_{\mathrm{MP}}^{-1}$ shall be extended by continuity to include the pole points in the CS range.

NOTE The CS generating function $\boldsymbol{G}_{\mathrm{MP}} = \boldsymbol{G}_{\mathrm{GD}} \circ \boldsymbol{Q}$ is not to be confused with the generating projection $\boldsymbol{P}$.

 

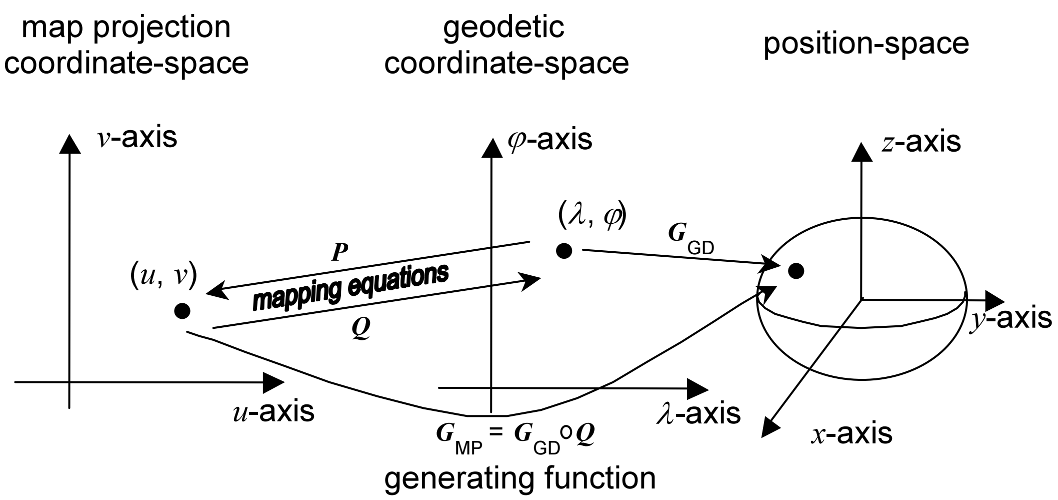


**Figure 5.4 — The generating function of a map projection**

### 5.8.3 Map projection geometry

#### 5.8.3.1 Introduction

In general, the Euclidean geometry that a surface CS 2D coordinate-space inherits from $\mathbf{R}^2$ has no direct significance with respect to the geometry of position-space. In particular, the Euclidean distance between a pair of surface geodetic coordinates has no obvious meaning in position-space. In contrast, map projections are specifically designed so that coordinate-space geometry will model one or more geometric aspects of the corresponding oblate ellipsoid surface in position-space.

The map projection CSs specified in this International Standard are designed so that one or more geometric aspects of the MP domain in the oblate ellipsoid surface are approximated or modelled by the corresponding aspect in coordinate-space. The length of the line segment between two map coordinates is related to the length of the corresponding surface curve. Similarly, one or more of directions, areas, the angles between two intersecting curves, and shapes may be related approximately or exactly to the corresponding geometric aspect on the oblate ellipsoid surface.

The extent to which these aspects are or are not closely related is an indication of distortion. Some map projection CSs are designed to eliminate distortion for one geometric aspect (such as angles or area). Others are designed to reduce distortion for several geometric aspects. In general, distortion tends to increase with the size of the oblate ellipsoid MP domain relative to the total oblate ellipsoid surface area. Conversely, distortion errors may be reduced by restricting the size of the MP domain. Map projections specified in this International Standard in the context of a spatial reference frame may have areas of definition beyond which the projection should not be used for some application domains due to unacceptable distortion[14].

#### 5.8.3.2 Conformal map projections

A *conformal map projection* preserves angles. For such map projections, when two surface curves on an oblate ellipsoid meet at the angle $\alpha$, the image of those curves in the map coordinate-space meet at the same angle $\alpha$ [THOM].

[14] It is a consequence of the *Theorema Egregium* of Gauss that no map projection CS can eliminate all distortion.

 

In addition, [THOM] contains a derivation based on the theory of complex variables to obtain conditions that specify when a projection is conformal. The map projections specified in Table 5.18 through Table 5.22 are conformal. The equidistant cylindrical MP specified in Table 5.23 is not conformal.

NOTE The conformal property is local. A conformal map projection preserves angles at a point, but does not necessarily preserve shape or area. In particular, a large projected triangle may appear distorted under a conformal map projection.

#### 5.8.3.3 Point distortion

One indicator of map projection length distortion is the ratio of lengths between an infinitesimal line segment in coordinate-space and the corresponding curve in position-space. Given a point in the interior of the MP range with surface geodetic coordinate $(\lambda,\varphi)$ the *directional point distortion*[15] at $(\lambda,\varphi)$ with respect to a smooth surface curve passing through the point is the ratio of the differential distance in coordinate-space to the differential arc length at $(\lambda,\varphi)$ along the curve as determined by the mapping equations.

The *latitudinal point distortion* at $(\lambda,\varphi)$, denoted $j(\lambda,\varphi)$, is the directional point distortion with respect to the meridian at $(\lambda,\varphi)$. It is computed in the direction of the meridian at the point as:

$$j(\lambda,\varphi) = \lim_{\Delta\to 0}\frac{\Delta(\text{arc length in coordinate-space})}{\Delta(\text{arc length along a meridian})} = \frac{\sqrt{(\partial u/\partial\varphi)^2 + (\partial v/\partial\varphi)^2}}{R_{\mathrm{M}}(\varphi)}$$

where $R_{\mathrm{M}}(\varphi)$ is the radius of curvature in the meridian as specified in Table 5.6.

The *longitudinal point distortion* at $(\lambda,\varphi)$, denoted $k(\lambda,\varphi)$, is the directional point distortion with respect to the parallel at $(\lambda,\varphi)$. It is computed in the direction of the parallel at the point as:

$$k(\lambda,\varphi) = \lim_{\Delta\to 0}\frac{\Delta(\text{arc length in coordinate-space})}{\Delta(\text{arc length along a parallel})} = \frac{\sqrt{(\partial u/\partial\lambda)^2 + (\partial v/\partial\lambda)^2}}{R_{\mathrm{N}}(\varphi)\cos(\varphi)}$$

where $R_{\mathrm{N}}(\varphi)$ is the radius of curvature in the prime vertical as specified in Table 5.6.

If a map projection is conformal, then the directional point distortion is independent of the direction of the curve at the point. In particular, $j(\lambda,\varphi) = k(\lambda,\varphi)$ for conformal map projections.

It is common practice in cartography to convert map projection coordinate-space to a display coordinate-space by means of a scaling factor. The scaling factor $\sigma$ shall be termed a *map scale* [HTDP] and a point in the display space shall be termed a *display coordinate*[16]. The relationship of a display coordinate $(u_{\mathrm{d}}, v_{\mathrm{d}})$ to a map coordinate $(u, v)$ is:

$$u_{\mathrm{d}} = \sigma u$$
$$v_{\mathrm{d}} = \sigma v$$

Map scale is commonly expressed as a ratio 1:$n$.

EXAMPLE A map scale printed on a map sheet as 1:50 000 corresponds to $\sigma$ = 1/50 000.

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15 This concept is found in the literature under a variety of names. The term “point distortion” is introduced to avoid ambiguity.

16 The distinction between a map projection coordinate and a display coordinate is not usually made explicit in the literature. The term “display coordinate” is introduced to avoid ambiguity.

For a conformal map projection, the infinitesimal ratio of display distance to arc length along a parallel is the *point scale* at $(\lambda, \varphi)$ and is denoted by $k_{\text{scaled}}$. The relationship between point scale and point distortion is:

$$k_{\text{scaled}}(\lambda, \varphi) = \sigma\, k(\lambda, \varphi).$$

#### 5.8.3.4 Geodetic azimuth and map azimuth

The *geodetic azimuth*[17] from a non-polar point $\boldsymbol{p}_1$ on the surface of an ellipsoid to a second point $\boldsymbol{p}_2$ on the surface is the angle measured clockwise from the meridian curve segment connecting $\boldsymbol{p}_1$ to the North pole to the geodesic containing $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ (see Figure 5.5). The range of azimuth values $\alpha$ shall be $0 \le \alpha < 2\pi$. The definition and range constraints apply to points in both hemispheres.

In a map projection CS, the *map azimuth* from a coordinate $\boldsymbol{c}_1$ to a coordinate $\boldsymbol{c}_2$ is defined as the angle from the $v$-axis (map-north) clockwise to the line segment connecting $\boldsymbol{c}_1$ to $\boldsymbol{c}_2$. In general, the map azimuth for a pair of coordinates will differ in value from the geodetic azimuth of the corresponding points on the oblate ellipsoid.

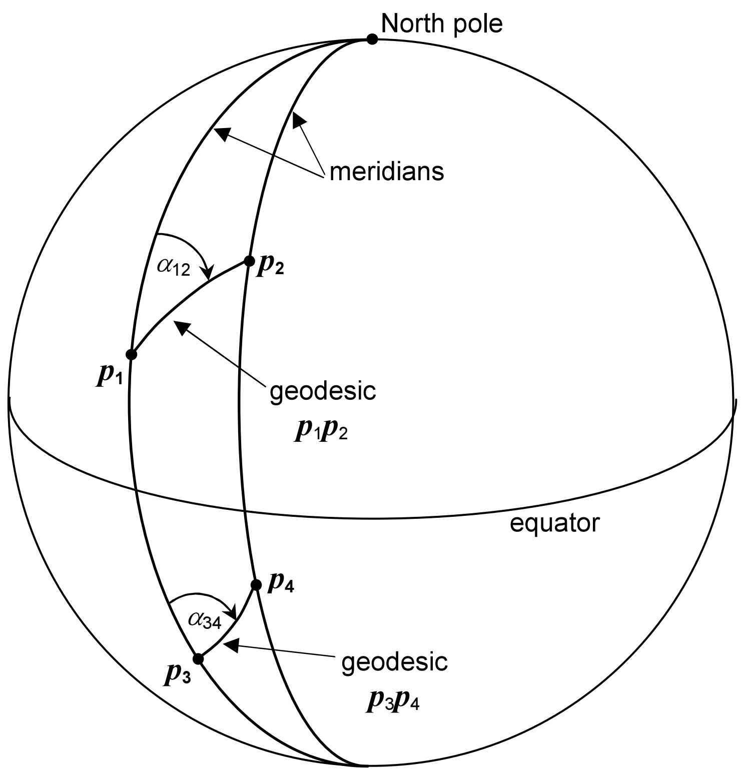


**Figure 5.5 — Geodetic azimuths $\alpha_{12}$ from $\boldsymbol{p}_1$ to $\boldsymbol{p}_2$ and $\alpha_{34}$ from $\boldsymbol{p}_3$ to $\boldsymbol{p}_4$**

[17] More general definitions that allow measurements of azimuth angle clockwise or counter-clockwise and from the north or south side of the meridian are in use. The generalization to the case for which one or more of the two points is not on the surface is treated in [RAPP1] and [RAPP2]. The more general definitions are not required for subsequent SRM concepts.

 

#### 5.8.3.5 Convergence of the meridian

Given a point $(\lambda, \varphi)$ in the interior of the MP domain of a map projection, the meridian through that point is projected to a curve in coordinate-space that passes through the corresponding coordinate. The angle $\gamma$ at the coordinate in the clockwise direction from the curve to the $v$-axis (map-north) direction shall be called the *convergence of the meridian* (COM) (see Figure 5.6).

The relationship $\gamma(\lambda, \varphi) = \operatorname{arctan2}\left(-\frac{\partial u}{\partial \varphi}, \frac{\partial v}{\partial \varphi}\right)$ is used to derive the formulae for COM from the mapping equations of each of the map projections[18]. The COM angle is adjusted to the range $-\pi < \gamma \leq \pi$.

NOTE If the map projection is conformal, then an equivalent relationship is given by: $\gamma(\lambda, \varphi) = \operatorname{arctan2}\left(\frac{\partial v}{\partial \lambda}, \frac{\partial u}{\partial \lambda}\right)$.

A typical geometry illustrating the COM at a point $\boldsymbol{p}$ is shown for the transverse Mercator map projection in Figure 5.6.

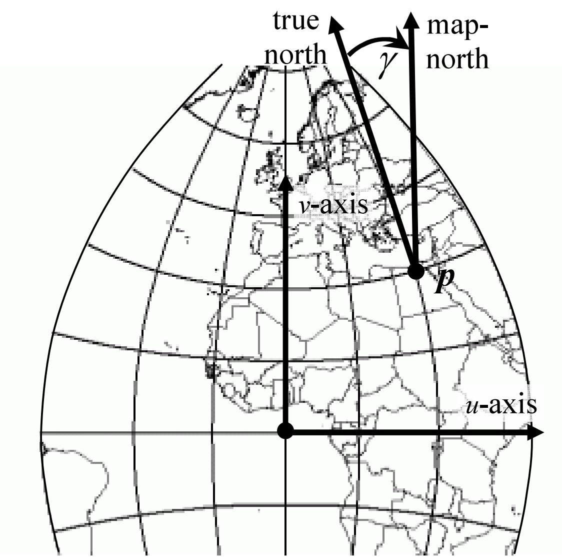


**Figure 5.6 — Convergence of the meridian**

EXAMPLE If $\boldsymbol{p}_2$ is directly map-north of $\boldsymbol{p}_1$ (it has a larger $v$ coordinate-component), then the map azimuth is zero, but the geodetic azimuth may not be zero. The geodetic azimuth is approximately the sum of the map azimuth and the COM if the points are sufficiently close together.

### 5.8.4 Relationship to projection functions

#### 5.8.4.1 Projection functions

Projection functions are defined in A.9. In some cases, the generating projection of a map projection CS is derived from a projection function. The derivation involves two steps. The first step is to restrict the domain of

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[18] The function arctan2 is defined in A.8.2.

the projection function to a specified region of a given oblate ellipsoid so that the restricted function is one-to-one. The range of a projection function is a surface in 3D position-space. The second step is to associate that surface to 2D coordinate-space without introducing additional distortions.

In the case of planar projection functions, including the orthographic, perspective, and stereographic projection functions, the range is in a plane that can be identified with 2D coordinate-space by selecting an origin and unit axis points.

In the case of the cylindrical and conic projection functions, the range surface is a cylinder or a cone, respectively. These surfaces are developable surfaces and, except for a line of discontinuity, are homeomorphic to a subset of 2D coordinate-space with a homeomorphism that has a Jacobian determinant equal to one. Conceptually, these surfaces can be unwrapped to a flat plane without stretching the surface.

The polar stereographic MP (Table 5.22) is derived from the stereographic projection function, and in the spherical case, it is a conformal map projection. The same derivation may be applied to an oblate ellipsoid. However, the resulting map projection will not have the conformal property. For this reason, the generalization of the polar stereographic map projection mapping equations from the spherical case to the non-spherical oblate ellipsoid case is not derived from the polar stereographic projection function. Instead it is derived analytically to preserve the conformal property. Similarly, the Mercator map projection (Table 5.18) is designed to have the conformal property and is not derived from the cylindrical projection function even in the case of a sphere.

EXAMPLE Polar stereographic: Given a sphere with a polar point $\boldsymbol{p}$, the tangent plane to the sphere at $\boldsymbol{p}$ and the opposite polar point $\boldsymbol{v}$ specify a stereographic planar projection function $\boldsymbol{F}$ (see A.9.2.3). The restriction of $\boldsymbol{F}$ to a subsurface of the sphere that excludes $\boldsymbol{v}$, is the generating projection for the sphere case of a polar stereographic map projection. In Figure 5.7 the position $\boldsymbol{s}$ on the sphere is projected to point $\boldsymbol{t}$ on a plane.

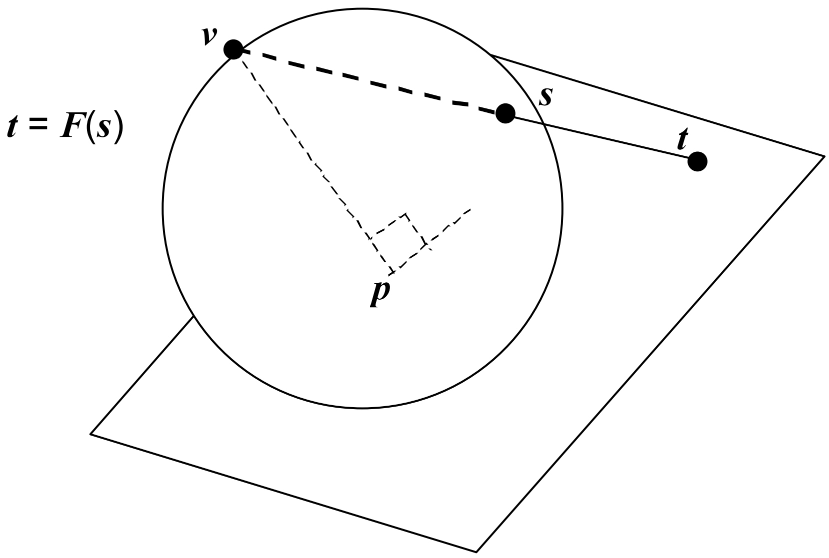



**Figure 5.7 — Polar stereographic map projection**

### 5.8.4.2 Map projection classification

The use of projection functions to derive map projections with desirable properties is limited, but does motivate some classifications of map projections. These classifications include tangent and secant map projections as well as conic and cylindrical map projections [SNYD, p. 5].

#### 5.8.4.3 Cylindrical map projections

A map projection is classified as *cylindrical* if:

a) all meridians of the oblate ellipsoid project to parallel straight lines that are equally-spaced with respect to the longitude of the meridians, and

b) all parallels of the oblate ellipsoid project to parallel straight lines that are perpendicular to the meridian images.

As a consequence, $\gamma(\lambda,\varphi)=0$ for a cylindrical map projection.

EXAMPLE The Mercator map projection (Table 5.18) and the equidistant cylindrical map projection (Table 5.23) are both classified as cylindrical map projections.

A cylindrical map projection is *tangent* if the longitudinal point distortion is equal to one along the equator. It is *secant* if the longitudinal point distortion is equal to one along two parallels equally-spaced from the equator in latitude. In that case, the parallel with positive latitude shall be called the *standard parallel*. Tangent and secant cylindrical map projections are illustrated in Figure 5.8.

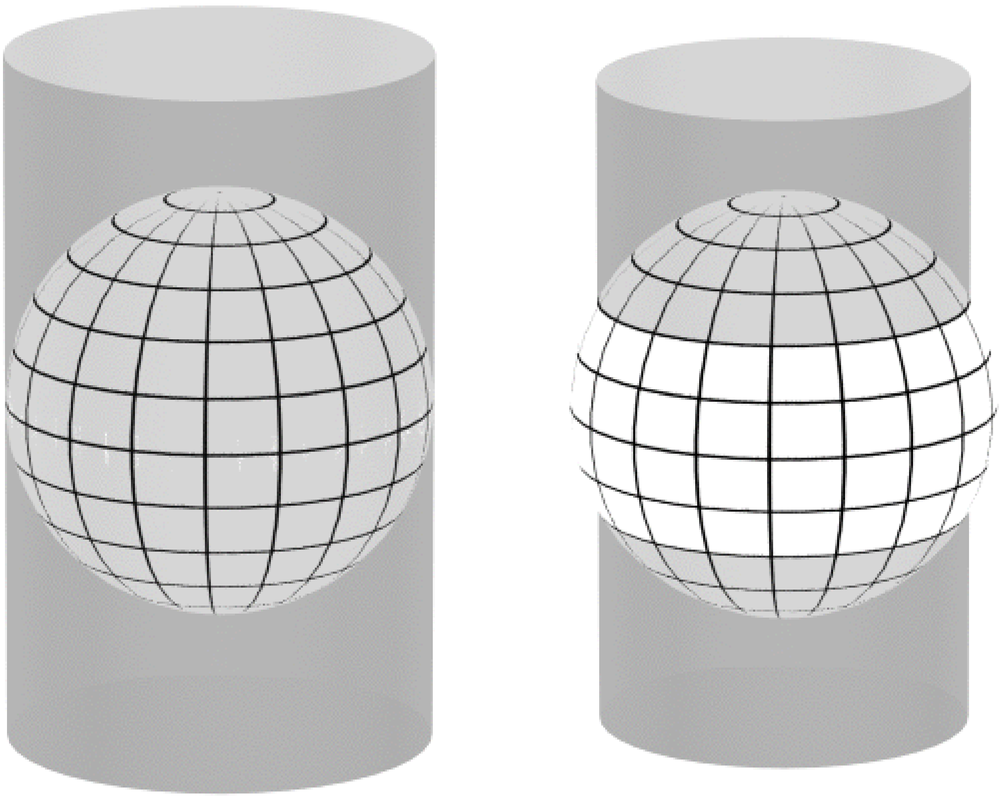


**Figure 5.8 — Tangent and secant cylindrical map projections**

#### 5.8.4.4 Conic map projections

A map projection is classified as *conic* if:

a) all meridians of the oblate ellipsoid project to radial straight lines that are equally-spaced in radial angle with respect to the longitude of the meridians, and

b) all parallels of the oblate ellipsoid project to concentric arcs that are perpendicular to the meridian images.

As a consequence, $\gamma$ depends only on $\lambda$ for conic map projections because the projections of meridians are straight line segments with an angle depending only on the longitude.

EXAMPLE Lambert conformal conic (see Table 5.21) is classified as a conic projection.

A conic map projection is *tangent* if along one parallel the longitudinal point distortion is equal to one. It is *secant* if the longitudinal point distortion is equal to one along two parallels that are not symmetric about the equator. In that case, the two parallels shall be called the *standard parallels* (see Figure 5.9).

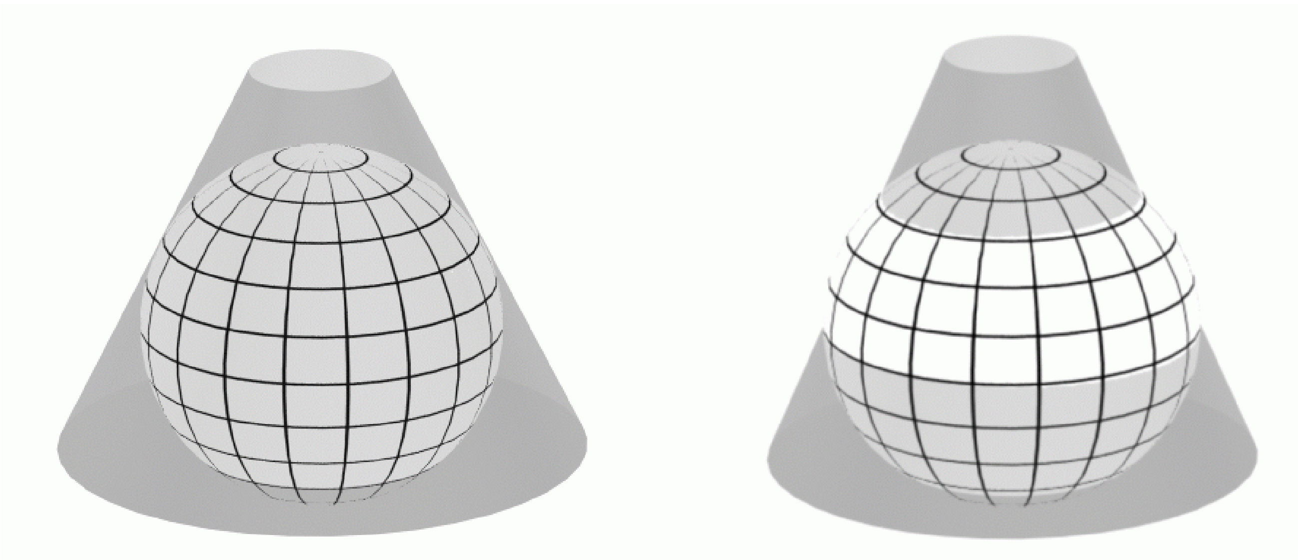


**Figure 5.9 — Tangent and secant conical map projections**

### 5.8.5 Map projection CS common parameters

To avoid negative coordinate-component values or to reduce the magnitude of the values in a region of interest in the coordinate-space of a map projection, two mapping equation parameters shall be provided to control the position of the coordinate-space origin $(0,0)$. One, denoted as $u_F$, shall be called the *false easting* and shall offset easting values. The second, denoted as $v_F$, shall be called the *false northing* and shall offset northing values.

A map projection CS specification may specify additional mapping equation parameters. The mapping equation parameters, including false easting and false northing shall be also be called the CS parameters.

The CS parameters *longitude of origin*, denoted by $\lambda_{\text{origin}}$, and *latitude of origin*, denoted by $\varphi_{\text{origin}}$, are historically associated with some map projection CSs. Typically, the position with map coordinate $(u_F, v_F)$ has geodetic longitude equal to $\lambda_{\text{origin}}$ and/or geodetic latitude equal to $\varphi_{\text{origin}}$.

If $\lambda_{\text{origin}}$ is present as a CS parameter in a map projection CS specification, it is used as the longitudinal centring function parameter (see $\Lambda_C$ in Table 5.6.)

The CS parameter *central scale*, denoted by $k_0$, when present, is intended to control the tangent/secant characteristics of the map projection CS and is therefore close to, but does not generally exceed, 1,0. In the case of a sphere, $k_0 = 1$ corresponds to a tangent projection, and $k_0 < 1$ corresponds to a secant projection. Typically, some point in the MP domain with geodetic longitude equal to $\lambda_{\text{origin}}$ and/or geodetic latitude equal to $\varphi_{\text{origin}}$ will have a longitudinal point distortion equal to $k_0$ .

For some MP cases, if $k_0 < 1$, there will exist a parallel with longitudinal point distortion equal to 1 at each point on the parallel. The latitude of such a parallel is termed a *secant latitude*, or a *latitude of true scale*.

NOTE Central scale should not be confused with Map scale. Map scales (see 5.8.3.3) are typically much smaller in magnitude and are applied directly to the coordinate-space. In particular, if a transverse Mercator map projection with central scale value $k_0$ = 0,9996 is to be scaled 1:50 000 on a map sheet display, then the mapping equations $P_1(\lambda,\ \varphi)$, and $P_2(\lambda,\ \varphi)$ are evaluated with $k_0$ = 0,9996 and the display coordinates $\left(\sigma P_1(\lambda,\ \varphi), \sigma P_2(\lambda,\ \varphi)\right)$, are plotted on the map sheet with $\sigma$ = (1/50 000).

### 5.8.6 Augmented map projections

#### 5.8.6.1 Augmentation with ellipsoidal height

A 3D CS can be specified from a map projection CS. The canonical embedding of a point $(u,\ v)$ in $\mathbf{R}^2$ to the point $(u,\ v,\ 0)$ in the $uv$-plane of $\mathbf{R}^3$ allows map points in 2D coordinate-space to be augmented with a third coordinate axis, the $w$-axis of $\mathbf{R}^3$. To be considered as a 3D CS, an augmented 3-tuple $(u,\ v,\ w)$ of coordinates in the *augmented map projection* coordinate-space shall be associated to a unique position in position-space. The association is to ellipsoidal height $h = w$. Given an augmented coordinate-tuple $(u,\ v,\ w)$ for which $(u,\ v)$ belongs to the coordinate range of the underlying generating projection, the associated position is given in 3D geodetic coordinates $(\lambda,\ \varphi,\ h)$ where $(\lambda,\ \varphi)$ is projected to $(u,\ v)$ by the map projection mapping equations. The third coordinate-space coordinate $w$ is the vertical coordinate and the 3D geodetic coordinate constraints on negative values of $h$ impose corresponding constraints on allowed values for $w$. In some application domains, other vertical coordinate measures are used (see Clause 9). Augmentation is restricted to ellipsoidal height in this International Standard.

#### 5.8.6.2 Distortion in augmented map projections

In addition to map projection distortion (see 5.8.3.1), augmentation causes additional distortion. Consider the two straight-line segments between the pairs of coordinate-space points $\{(u_1,\ v_1,\ 0),\ (u_2,\ v_2,\ 0)\}$ and $\{(u_1,\ v_1,\ w),\ (u_2,\ v_2,\ w)\}$ with $w > 0$ (see Figure 5.10). In augmented map projection geometry, the two line segments have the same length. The corresponding curve in position-space of the first line segment is a surface curve of the oblate ellipsoid (or sphere). The corresponding second curve is outside of the oblate ellipsoid (or sphere) and has longer arc length than the first, and the length difference increases with $w$.

In general, vertical angles are not preserved and the angular error will vary with the point distortion value. These and other distortions have profound implications for dynamic equations that are beyond the scope of this International Standard.

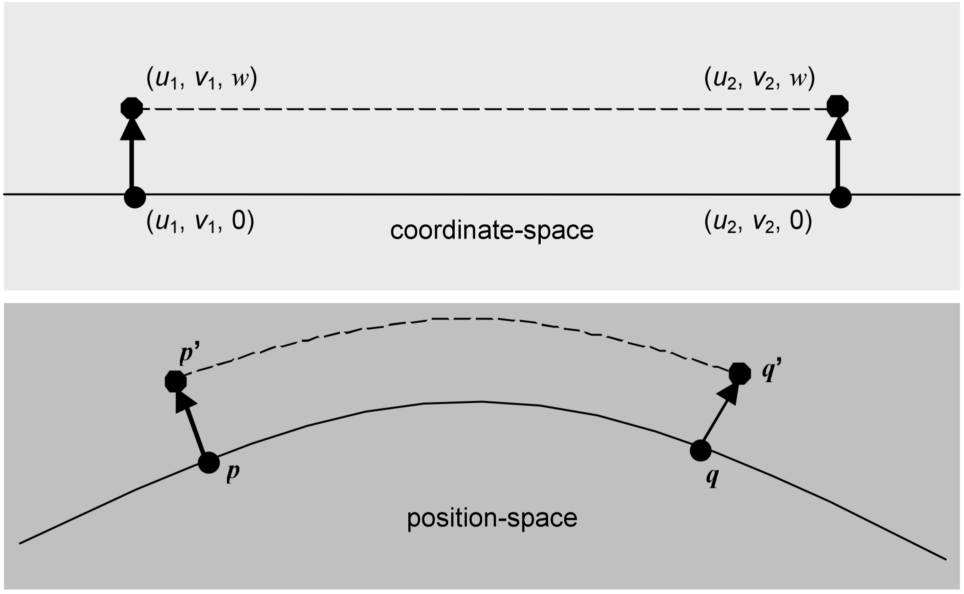



**Figure 5.10 — Vertical distortion**

## 5.9 CS specifications

### 5.9.1 Specification table elements and common functions and parameters

The CSs specified in this International Standard are presented in Table 5.8 through Table 5.35. Each CS specification specifies the values of all elements presented in Table 5.5.

**Table 5.5 — Coordinate system specification elements**

| Element | Definition |
|---|---|
| **Description** | A description of the CS including a common name, if any. |
| **CS label** | The label of the CS (see 13.2.2). |
| **CS code** | The code of the CS (see 13.2.3). Code 0 (UNSPECIFIED) is reserved. |
| **Function type** | Either "generating function" or "map projection". |
| **CS descriptor** | One of: 3D linear, 3D curvilinear, surface linear, surface curvilinear, map projection, 2D linear, 2D curvilinear, 1D linear, 1D curvilinear, or surface (map projection) and 3D (augmented map projection). |
| **Properties** | Either "none" or a list of one or more properties of the CS chosen from the following: orthogonal, not orthogonal, orthonormal, not orthonormal, conformal, or not conformal. Conformal and not conformal only apply to map projections. |
| **CS parameters and constraints** | The CS parameters (if any) along with any constraints on how those parameters interrelate, otherwise "none". |
| **Coordinate-components** | Coordinate-component symbols and common names in a specified order. |
| **Domain of the generating function or mapping equations** | The domain of the CS generating function or the mapping equations. |
| **Generating function or mapping equations** | The CS generating function or mapping equations. |

| Element | Definition |
|---|---|
| **Domain of the inverse of the generating function or mapping equations** | The domain of the inverse of the CS generating function or the domain of the inverse of mapping equations. |
| **Inverse of the generating function or mapping equations** | The inverse of the CS generating function or the inverse of mapping equations. |
| **COM** | For map projection CSs, the equation for $\gamma$ in radians. Otherwise "n/a". |
| **Point distortion** | For map projection CSs, the equation for $k$, if conformal, or the equations for $k$ and $j$, if non-conformal. Otherwise "n/a". |
| **Figures** | Zero or more figure(s) that explain and illustrate the CS. |
| **Notes** | Optional, non-normative information concerning the CS, otherwise "none". |
| **References** | The references (see 13.2.5). |

A specific ordering of coordinate-components for a coordinate $n$-tuple in a CS specification is required in this International Standard for clarity of presentation, to avoid ambiguity in the specification of the API, and, in the case of an orthogonal CS of CS type 3D, to ensure the right-handed CS property (see 5.6.4). Coordinate values may be represented using any of the methods delineated in 8.5.1.

Several specified CS generating functions and mapping equations and/or their inverses use some common intermediate functions or parameters associated with oblate ellipsoids. For clarity and concise presentation, these functions and parameters are defined in Table 5.6.

**Table 5.6 — Common parameters and functions of an oblate ellipsoid**

| Function or parameter | Symbol and defining expression |
|---|---|
| major semi-axis | $a$ |
| minor semi-axis | $b$ |
| flattening | $f = 1 - \frac{b}{a}$ |
| (first) eccentricity | $\varepsilon^2 = 1 - \left(\frac{b}{a}\right)^2$<br>alternative equivalent expression:<br>$\varepsilon^2 = 2f - f^2$ |
| second eccentricity | $(\varepsilon')^2 = \frac{\varepsilon^2}{(1-\varepsilon^2)}$ |
| radius of curvature in the prime vertical | $R_N(\varphi) = \frac{a}{\sqrt{1-\varepsilon^2 \sin^2 \varphi}}$ |

| Function or parameter | Symbol and defining expression |
|---|---|
| radius of curvature in the meridian | $R_M(\varphi) = \dfrac{a(1-\varepsilon^2)}{\left(\sqrt{1-\varepsilon^2 \sin^2\varphi}\right)^3}$ <br> alternative equivalent expression: <br> $R_M(\varphi) = \dfrac{(1-\varepsilon^2)}{a^2}\left(R_N(\varphi)\right)^3$ |
| meridional distance to equator | $S(\varphi) = \int_0^{\varphi} R_M(\xi)\, d\xi$ |
| longitudinal centring about $\lambda_C$ | $\Lambda_C(\lambda, \lambda_C) = \begin{cases} \lambda - \lambda_C & \text{if } -\pi < \lambda - \lambda_C \le \pi \\ \lambda - \lambda_C - 2\pi & \text{if } \pi < \lambda - \lambda_C \\ \lambda - \lambda_C + 2\pi & \text{if } \lambda - \lambda_C \le -\pi \end{cases}$ |

NOTE 1 Replacing $\lambda_C$ with $-\lambda_C$ gives the inverse of the longitudinal centring function. That is:

if $\Lambda^* = \Lambda_C(\lambda, \lambda_C)$, then $\lambda = \Lambda_C(\Lambda^*, -\lambda_C)$.

NOTE 2 The function arctan2, used in many CS specification tables, is defined in A.8.2.

Table 5.7 presents a directory of the CS specifications in this International Standard. Each listed CS is specified in a separate table that is indicated by the hyperlink in the corresponding cell in the “Table number” column. Additional CSs may be added by registration (see Clause 13).

**Table 5.7 — CS specification directory**

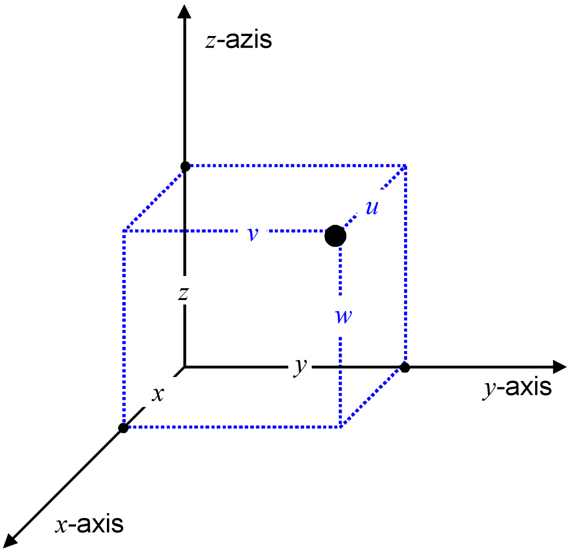
| Function type | CS type | Label and description | Table number |
|---|---|---|---|
| Generating function | 3D | EUCLIDEAN_3D<br>Euclidean 3D | Table 5.8 |
| | | LOCOCENTRIC_EUCLIDEAN_3D<br>Lococentric Euclidean 3D | Table 5.9 |
| | | EQUATORIAL_SPHERICAL<br>Equatorial spherical | Table 5.10 |
| | | LOCOCENTRIC_EQUATORIAL_SPHERICAL<br>Lococentric equatorial spherical | Table 5.11 |
| | | AZIMUTHAL_SPHERICAL<br>Azimuthal spherical | Table 5.12 |
| | | LOCOCENTRIC_AZIMUTHAL_SPHERICAL<br>Lococentric azimuthal spherical | Table 5.13 |
| | | GEODETIC<br>Geodetic 3D | Table 5.14 |
| | | PLANETODETIC<br>Planetodetic 3D | Table 5.15 |

| Function type | CS type | Label and description | Table number |
|---|---|---|---|
| | | CYLINDRICAL<br>Cylindrical | Table 5.16 |
| | | LOCOCENTRIC_CYLINDRICAL<br>Lococentric cylindrical | Table 5.17 |
| Map projection | Surface and augmented 3D | MERCATOR<br>Mercator | Table 5.18 |
| | | OBLIQUE_MERCATOR_SPHERICAL<br>Oblique Mercator spherical | Table 5.19 |
| | | TRANSVERSE_MERCATOR<br>Transverse Mercator | Table 5.20 |
| | | LAMBERT_CONFORMAL_CONIC<br>Lambert conformal conic | Table 5.21 |
| | | POLAR_STEREOGRAPHIC<br>Polar stereographic | Table 5.22 |
| | | EQUIDISTANT_CYLINDRICAL<br>Equidistant cylindrical | Table 5.23 |
| Generating function | Surface | SURFACE_GEODETIC<br>Surface geodetic | Table 5.24 |
| | | SURFACE_PLANETODETIC<br>Surface planetodetic | Table 5.25 |
| | | LOCOCENTRIC_SURFACE_EUCLIDEAN<br>Lococentric surface Euclidean | Table 5.26 |
| | | LOCOCENTRIC_SURFACE_AZIMUTHAL<br>Lococentric surface azimuthal | Table 5.27 |
| | | LOCOCENTRIC_SURFACE_POLAR<br>Lococentric surface polar | Table 5.28 |
| | 2D | EUCLIDEAN_2D<br>Euclidean 2D | Table 5.29 |
| | | LOCOCENTRIC_EUCLIDEAN_2D<br>Lococentric Euclidean 2D | Table 5.30 |
| | | AZIMUTHAL<br>Azimuthal | Table 5.31 |
| | | LOCOCENTRIC_AZIMUTHAL<br>Lococentric azimuthal | Table 5.32 |
| | | POLAR<br>Polar | Table 5.33 |
| | | LOCOCENTRIC_POLAR<br>Lococentric polar | Table 5.34 |
| | 1D | EUCLIDEAN_1D<br>Euclidean 1D | Table 5.35 |

### 5.9.2 Euclidean 3D CS specification

**Table 5.8 — Euclidean 3D CS**

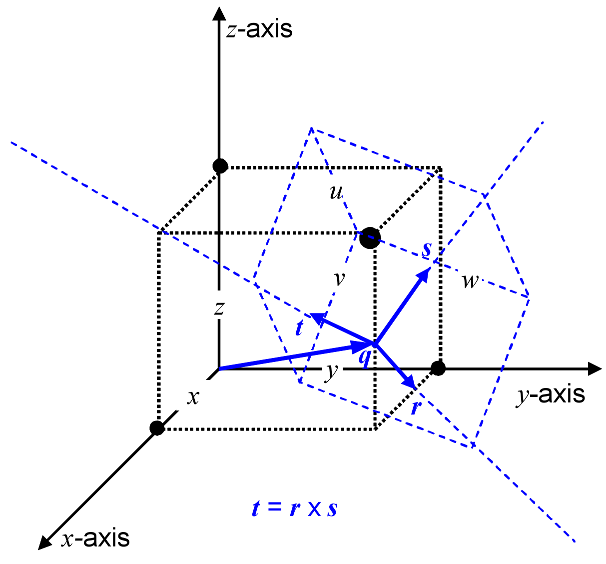
| Element | Specification |
|---|---|
| **Description** | Euclidean 3D. |
| **CS label** | EUCLIDEAN_3D |
| **CS code** | 1 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D linear. |
| **Properties** | Orthonormal. |
| **CS parameters and constraints** | none |
| **Coordinate-components** | $u$, $v$, $w$ |
| **Domain of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Generating function or mapping equations** | $\boldsymbol{F}_{\text{E3D}}(u,v,w)=(x,y,z)$,<br>where:<br>$x = u$,<br>$y = v$, and<br>$z = w$. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}_{\text{E3D}}^{-1}(x,y,z)=(u,v,w)$,<br>where:<br>$u = x$,<br>$v = y$, and<br>$w = z$. |
| **COM** | n/a |
| **Point distortion** | n/a |

| Element | Specification |
|---|---|
| Figures |  |
| Notes | Coordinate-space 3-tuples are identified with position-space 3-tuples. |
| References | [EDM] |

### 5.9.3 Lococentric Euclidean 3D CS specification

**Table 5.9 — Lococentric Euclidean 3D CS**

| Element | Specification |
|---|---|
| **Description** | Localized Euclidean 3D |
| **CS label** | LOCOCENTRIC_EUCLIDEAN_3D |
| **CS code** | 2 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D linear. |
| **Properties** | Orthonormal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal vectors. |
| **Coordinate-components** | $u$, $v$, $w$ |
| **Domain of the generating function or mapping equations** | $\mathbf{R}^3$ |

<table>
<tr><th>Element</th><th>Specification</th></tr>
<tr><td>Generating function or mapping equations</td><td>$\boldsymbol{F}(u,v,w) = \boldsymbol{L}_{\mathrm{3D}} \circ \boldsymbol{F}_{\mathrm{E3D}}(u,v,w)$,<br>where:<br>$\boldsymbol{L}_{\mathrm{3D}}$ = the 3D localization operator, and<br>$\boldsymbol{F}_{\mathrm{E3D}}$ = the Euclidean 3D CS generating function.</td></tr>
<tr><td>Domain of the inverse of the generating function or mapping equations</td><td>$\mathbf{R}^3$</td></tr>
<tr><td>Inverse of the generating function or mapping equations</td><td>$\boldsymbol{F}^{-1}(x,y,z) = \boldsymbol{F}_{\mathrm{E3D}}^{-1} \circ \boldsymbol{L}_{\mathrm{3D}}^{-1}(x,y,z)$,<br>where:<br>$\boldsymbol{L}_{\mathrm{3D}}^{-1}$ = the 3D localization inverse operator, and<br>$\boldsymbol{F}_{\mathrm{E3D}}^{-1}$ = the Euclidean 3D CS inverse generating function.</td></tr>
<tr><td>COM</td><td>n/a</td></tr>
<tr><td>Point distortion</td><td>n/a</td></tr>
<tr><td>Figures</td><td>
</td></tr>
<tr><td>Notes</td><td>1) Euclidean 3D CS (see Table 5.8) is a special case with $\boldsymbol{q} = (0,0,0)$, $\boldsymbol{r} = (1,0,0)$, $\boldsymbol{s} = (0,1,0)$.<br><br>2) The generating function is the composition of the generating function for Euclidean 3D CS (see Table 5.8) with the 3D localization operator (see 5.7).</td></tr>
<tr><td>References</td><td>[EDM]</td></tr>
</table>

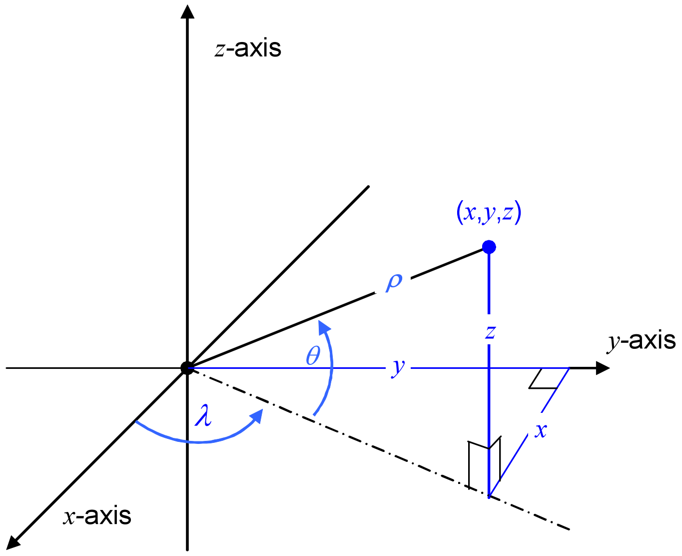
### 5.9.4 Equatorial spherical CS specification

**Table 5.10 — Equatorial spherical CS**

| Element | Specification |
|---|---|
| **Description** | Equatorial spherical. |

| Element | Specification |
|---|---|
| **CS label** | EQUATORIAL_SPHERICAL |
| **CS code** | 3 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | none |
| **Coordinate-components** | $\lambda$: longitude in radians,<br>$\theta$: spherical latitude in radians, and<br>$\rho$: radius. |
| **Domain of the generating function or mapping equations** | $\left\{(\lambda,\theta,\rho) \text{ in } \mathbf{R}^3 \mid -\pi < \lambda \le \pi,\ -\pi/2 < \theta < \pi/2, \text{ and } 0 < \rho\right\}$<br>$\cup \left\{(0,\theta,\rho) \text{ in } \mathbf{R}^3 \mid \theta = \pm\pi/2,\ \rho > 0\right\} \cup \left\{(0,0,0)\right\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}_{\mathrm{S}}(\lambda,\theta,\rho) = (x,y,z)$,<br>where:<br>$x = \rho\cos(\theta)\cos(\lambda)$,<br>$y = \rho\cos(\theta)\sin(\lambda)$, and<br>$z = \rho\sin(\theta)$. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}_{\mathrm{S}}^{-1}(x,y,z) = (\lambda,\theta,\rho)$,<br>where:<br>$\lambda = \operatorname{arctan2}(y,x)$,<br>$\theta = \begin{cases} \arcsin(z/\rho) \text{ (principal value)}, & \text{if } \rho > 0 \\ 0, & \text{if } \rho = 0 \end{cases}$<br>and:<br>$\rho = \sqrt{x^2 + y^2 + z^2}$. |
| **COM** | n/a |
| **Point distortion** | n/a |

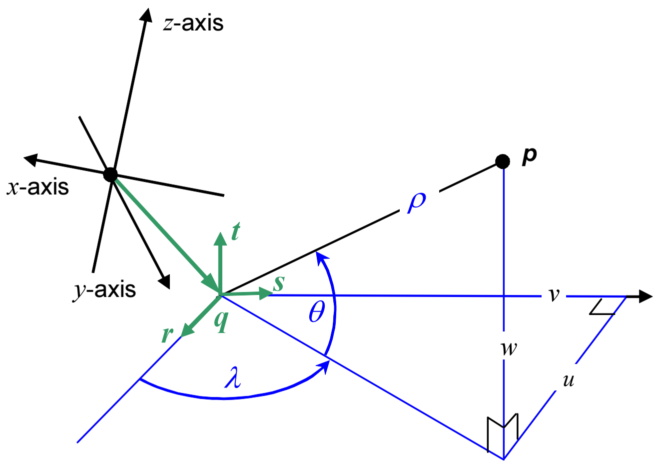
 

| Element | Specification |
|---|---|
| **Figures** |  |
| **Notes** | 1) The equatorial spherical CS is not intrinsically associated with any specific sphere.<br>2) In many application domains, the co-latitude $\psi = \pi/2 - \theta$ is used. The spherical latitude $\theta$ has been specified for compatibility with astronomical declination. The modifier "equatorial" is used to emphasize this difference.<br>3) The inverse generation function is discontinuous on the $z$-axis. |
| **References** | [HCP] |

### 5.9.5 Lococentric equatorial spherical CS specification

**Table 5.11 — Lococentric equatorial spherical CS**

| Element | Specification |
|---|---|
| **Description** | Localized equatorial spherical |
| **CS label** | LOCOCENTRIC_EQUATORIAL_SPHERICAL |
| **CS code** | 4 |
| **Function type** | Generating function |
| **CS descriptor** | 3D curvilinear |
| **Properties** | Orthogonal |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$: the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$: axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |

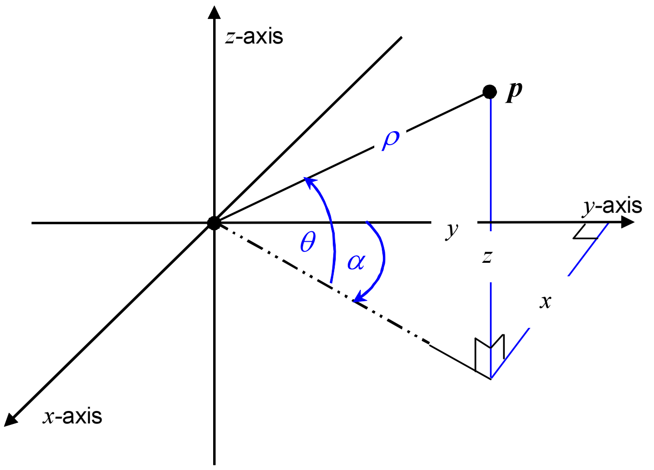
| Element | Specification |
|---|---|
| **Coordinate-components** | $\lambda$ : longitude in radians,<br>$\theta$ : spherical latitude in radians, and<br>$\rho$ : radius. |
| **Domain of the generating function or mapping equations** | $\{(\lambda,\theta,\rho) \text{ in } \mathbf{R}^3 \mid -\pi < \lambda \le \pi, \; -\pi/2 < \theta < \pi/2, \text{ and } 0 < \rho\}$<br>$\cup \{(0,\theta,\rho) \text{ in } \mathbf{R}^3 \mid \theta = \pm\pi/2, \; \rho > 0\} \cup \{(0,0,0)\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\lambda,\theta,\rho) = \boldsymbol{L}_{3D} \circ \boldsymbol{F}_{S}(\lambda,\theta,\rho)$,<br>where:<br>$\boldsymbol{L}_{3D}$ = the 3D localization operator, and<br>$\boldsymbol{F}_{S}$ = the spherical CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = \boldsymbol{F}_{S}^{-1} \circ \boldsymbol{L}_{3D}^{-1}(x,y,z)$,<br>where:<br>$\boldsymbol{L}_{3D}^{-1}$ = the 3D localization inverse operator, and<br>$\boldsymbol{F}_{S}^{-1}$ = the spherical CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |   |
| **Notes** | The generating function is the composition of the generating function for equatorial spherical CS (see Table 5.10) with the 3D localization operator (see 5.7). |
| **References** | [EDM] |

#### 5.9.6 Azimuthal spherical CS specification

**Table 5.12 — Azimuthal spherical CS**

| Element | Specification |
|---|---|

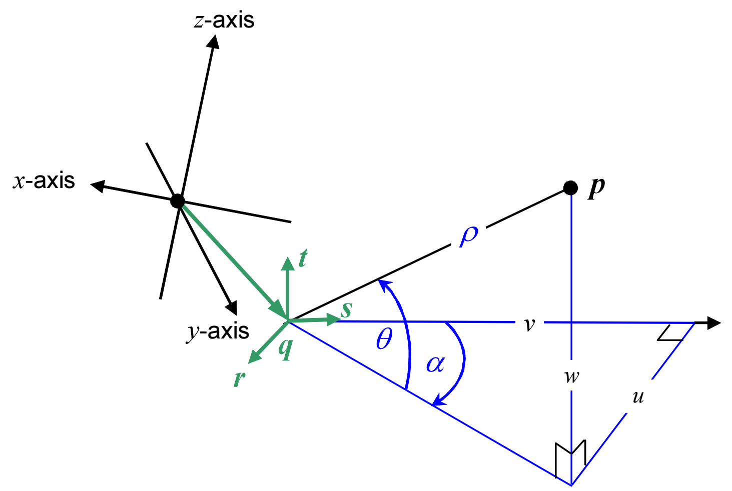
| Element | Specification |
|---|---|
| **Description** | Azimuthal spherical. |
| **CS label** | AZIMUTHAL_SPHERICAL |
| **CS code** | 5 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | none |
| **Coordinate-components** | $\alpha$ : azimuth in radians,<br>$\rho$ : radius, and<br>$\theta$ : depression/elevation angle in radians. |
| **Domain of the generating function or mapping equations** | $\left\{(\alpha,\rho,\theta) \text{ in } \mathbf{R}^3 \;\middle|\; 0 \le \alpha < 2\pi,\ 0 < \rho, \text{ and } -\pi/2 < \theta < \pi/2\right\}$<br>$\cup \left\{(0,\rho,\theta) \text{ in } \mathbf{R}^3 \;\middle|\; 0 < \rho,\ \theta = \pm\pi/2\right\} \cup \left\{(0,0,0)\right\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\alpha,\rho,\theta) = (x,y,z)$,<br>where:<br>$x = \rho\cos(\theta)\sin(\alpha)$,<br>$y = \rho\cos(\theta)\cos(\alpha)$, and<br>$z = \rho\sin(\theta)$. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = (\alpha,\rho,\theta)$,<br>where:<br>$\alpha = \begin{cases} \alpha' & \text{if } \alpha' \ge 0 \\ 2\pi + \alpha' & \text{if } \alpha' < 0 \end{cases}$,<br>$\alpha' = \operatorname{arctan2}(x,y)$<br>$\rho = \sqrt{x^2+y^2+z^2}$, and<br>$\theta = \begin{cases} \arcsin(z/\rho) \text{ (principal value)}, & \text{if } \rho > 0 \\ 0, & \text{if } \rho = 0. \end{cases}$ |
| **COM** | n/a |
| **Point distortion** | n/a |

| Element | Specification |
|---|---|
| **Figures** |   |
| **Notes** | 1) The inverse generating function is discontinuous on the *z*-axis.<br>2) The commonly used coordinate-component orderings are either $(\rho,\alpha,\theta)$ or $(\alpha,\theta,\rho)$. The coordinate-component ordering has been specified as $(\alpha,\rho,\theta)$ to ensure that this CS is right-handed. Compliant coordinate value representations are delineated in 8.5.1. |
| **References** | [EDM] |

### 5.9.7 Lococentric azimuthal spherical CS specification

**Table 5.13 — Lococentric azimuthal spherical CS**

| Element | Specification |
|---|---|
| **Description** | Localized azimuthal spherical. |
| **CS label** | LOCOCENTRIC_AZIMUTHAL_SPHERICAL |
| **CS code** | 6 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $\alpha$ : azimuth in radians,<br>$\rho$ : radius, and<br>$\theta$ : depression/elevation angle in radians. |
| **Domain of the generating function or mapping equations** | $\{(\alpha,\rho,\theta) \text{ in } \mathbf{R}^3 \mid 0 \le \alpha < 2\pi,\ 0 < \rho, \text{ and } -\pi/2 < \theta < \pi/2\}$<br>$\cup \{(0,\rho,\theta) \text{ in } \mathbf{R}^3 \mid 0 < \rho,\ \theta = \pm\pi/2\} \cup \{(0,0,0)\}$ |

| Element | Specification |
|---|---|
| **Generating function or mapping equations** | $F(\alpha,\rho,\theta) = L_{3D} \circ F_{AS}(\alpha,\rho,\theta)$,<br>where:<br>$L_{3D}$ = the 3D localization operator, and<br>$F_{AS}$ = the azimuthal spherical CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Inverse of the generating function or mapping equations** | $F^{-1}(x,y,z) = F_{AS}^{-1} \circ L_{3D}^{-1}(x,y,z)$,<br>where:<br>$L_{3D}^{-1}$ = the 3D localization inverse operator, and<br>$F_{AS}^{-1}$ = the azimuthal spherical CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |
| **Notes** | 1) The generating function is the composition of the generating function for azimuthal spherical (see Table 5.12) with the 3D localization operator (see 5.7).<br>2) The commonly used coordinate-component orderings are either $(\rho,\alpha,\theta)$ or $(\alpha,\theta,\rho)$. The coordinate-component ordering has been specified as $(\alpha,\rho,\theta)$ to ensure that this CS is right-handed. Compliant coordinate value representations are delineated in 8.5.1.<br>3) The inverse generating function is discontinuous on the $z$-axis. |
| **References** | [EDM] |

#### 5.9.8 Geodetic 3D CS specification

**Table 5.14 — Geodetic 3D CS**

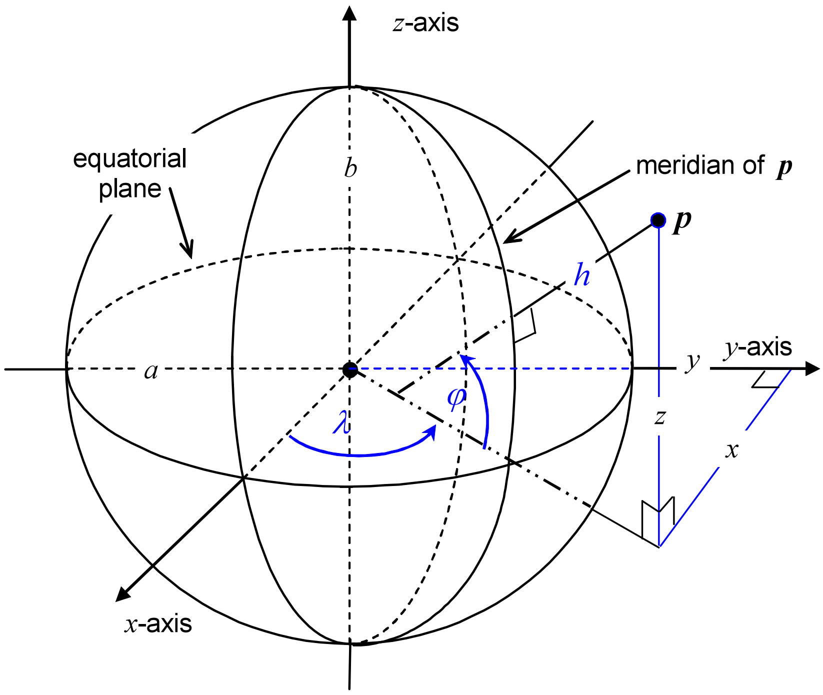
| Element | Specification |
|---|---|

| Element | Specification |
|---|---|
| **Description** | Geodetic 3D. |
| **CS label** | GEODETIC |
| **CS code** | 7 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | $a$: major semi-axis length<br>$b$: minor semi-axis length<br>Constraints:<br>$a > b$: (oblate ellipsoid)<br>$a = b$: (sphere) |
| **Coordinate-components** | $\lambda$ : longitude in radians,<br>$\varphi$ : geodetic latitude in radians, and<br>$h$ : ellipsoidal height. |
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \leq \pi$<br>$-\pi/2 < \varphi < \pi/2$<br>$-b < h$<br>and $(0, \pm\pi/2, h)$. |
| **Generating function or mapping equations** | $\boldsymbol{F}(\lambda,\varphi,h) = (x,y,z)$,<br>where:<br>$x = \left(R_{\mathrm{N}}(\varphi) + h\right)\cos(\varphi)\cos(\lambda)$,<br>$y = \left(R_{\mathrm{N}}(\varphi) + h\right)\cos(\varphi)\sin(\lambda)$, and<br>$z = \left((1-\varepsilon^2)R_{\mathrm{N}}(\varphi) + h\right)\sin(\varphi)$.<br>Simplification if $a = b$:<br>$\boldsymbol{F}(\lambda,\varphi,h) = (x,y,z)$,<br>where:<br>$x = (a+h)\cos(\varphi)\cos(\lambda)$,<br>$y = (a+h)\cos(\varphi)\sin(\lambda)$, and<br>$z = (a+h)\sin(\varphi)$. |
| **Domain of the inverse of the generating function or mapping equations** | $(a-b) < x^2 + y^2 + z^2$ |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z)=(\lambda,\varphi,h)$,<br>where:<br>$\lambda = \text{arctan}2(y,\, x)$,<br>$\varphi = \text{arctan}2\left(\left(z+\varepsilon'^2 z_0\right),\, w\right)$,<br>$h = u\left(1-\dfrac{b^2}{av}\right)$,<br>$w = \left(x^2+y^2\right)^{\frac{1}{2}}$,<br>$z_0 = \dfrac{b^2 z}{av}$,<br>$u = \left(\left(w-\varepsilon^2 w_0\right)^2 + z^2\right)^{\frac{1}{2}}$,<br>$v = \left(\left(w-\varepsilon^2 w_0\right)^2 + \left(1-\varepsilon^2\right) z^2\right)^{\frac{1}{2}}$,<br>$w_0 = \dfrac{-p\varepsilon^2 w}{(1+q)} + \left[\dfrac{a^2}{2}\left(1+\dfrac{1}{q}\right) - \dfrac{p\left(1-\varepsilon^2\right)z^2}{q(1+q)} - \dfrac{pw^2}{2}\right]^{\frac{1}{2}}$,<br>$q = \left(1+2\varepsilon^4 p\right)^{\frac{1}{2}}$,<br>$p = F\Big/\left(3\left(s+\dfrac{1}{s}+1\right)^2 G^2\right)$,<br>$s = \left[1+d+\left(d^2+2d\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$,<br>$d = \varepsilon^4 F w^2 / G^3$,<br>$G = w^2 + \left(1-\varepsilon^2\right)z^2 - \varepsilon^2\left(a^2-b^2\right)$, and<br>$F = 54 b^2 z^2$.<br>Simplification if $a=b$:<br>$\varphi = \text{arcsin}\left(z/(h+a)\right)$ (principal value), and<br>$h = \sqrt{x^2+y^2+z^2} - a$. |
| **COM** | n/a |
| **Point distortion** | n/a |

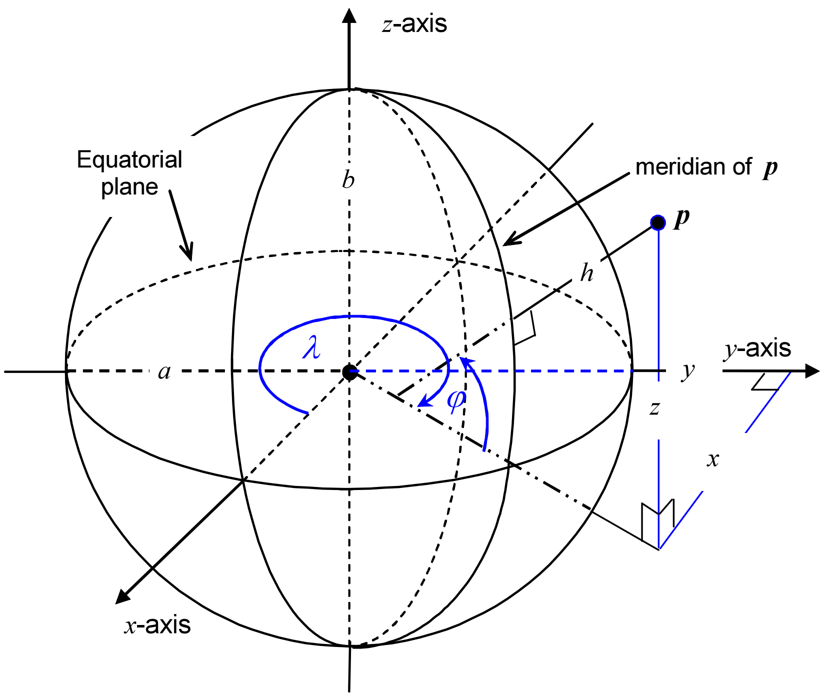
 

| Element | Specification |
|---|---|
| Figures |  |
| Notes | 1) The surface geodetic CS (Table 5.24) is induced on the 3rd coordinate-component surface at any point for which $h$ = 0.<br>2) If $a = b$, the geodetic latitude $\varphi$ coincides with the spherical latitude $\theta$ (see Table 5.10).<br>3) The inverse generating function is not continuous on the oblate ellipsoid rotational axis. |
| References | [HEIK] |

### 5.9.9 Planetodetic 3D specification

**Table 5.15 — Planetodetic CS**

| Element | Specification |
|---|---|
| Description | Planetodetic 3D. Geodetic 3D with longitude in opposite direction. |
| CS label | PLANETODETIC |
| CS code | 8 |
| Function type | Generating function. |
| CS descriptor | 3D curvilinear. |
| Properties | Orthogonal. |
| CS parameters and constraints | $a$: major semi-axis length<br>$b$: minor semi-axis length<br>Constraints:<br>$a > b$: (oblate ellipsoid)<br>$a = b$: (sphere) |

| Element | Specification |
|---|---|
| **Coordinate-components** | $\varphi$ : geodetic latitude in radians,<br>$\lambda$ : planetodetic longitude in radians, and<br>$h$ : ellipsoidal height. |
| **Domain of the generating function or mapping equations** | $-\pi/2 < \varphi < \pi/2$<br>$-\pi < \lambda \le \pi$<br>and $(0, \pm\pi/2, h)$. |
| **Generating function or mapping equations** | $\boldsymbol{F}(\varphi,\lambda,h) = \boldsymbol{F}_{\mathrm{GD}}(-\lambda,\varphi,h)$,<br>where:<br>$\boldsymbol{F}_{\mathrm{GD}}$ is the geodetic 3D generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $(a-b) < x^2 + y^2 + z^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = (\varphi,\lambda,h)$,<br>where:<br>$(\lambda,\varphi,h) = \boldsymbol{F}_{\mathrm{GD}}^{-1}(x,-y,z)$, and<br>$\boldsymbol{F}_{\mathrm{GD}}^{-1}$ is the geodetic 3D inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** | <br> |

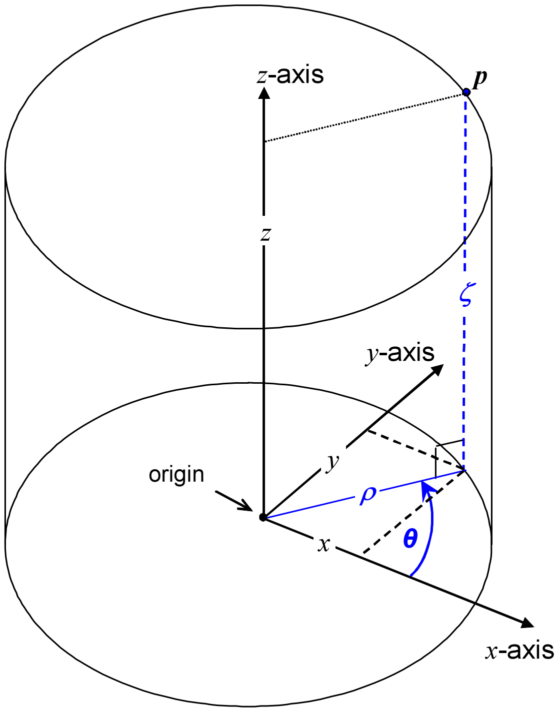
| Element | Specification |
|---|---|
| **Notes** | 1) Similar to the geodetic 3D CS (see Table 5.14) except that longitude increases in the opposite direction. In particular, points on a planet surface rotating (prograde) into view have larger planetodetic longitudes than those points rotating out of view. This CS is also called planetocentric when $a = b$ and planetographic when $a > b$.<br>2) The inverse generating function is not continuous on the oblate ellipsoid rotational axis.<br>3) The coordinate-component ordering differs from that of geodetic 3D CS to satisfy the right handedness requirement. |
| **References** | [RIIC] |

### 5.9.10 Cylindrical CS specification

**Table 5.16 — Cylindrical CS**

| Element | Specification |
|---|---|
| **Description** | Cylindrical. |
| **CS label** | CYLINDRICAL |
| **CS code** | 9 |
| **Function type** | Generating function. |
| **CS descriptor** | 3D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | none |
| **Coordinate-components** | $\rho$: radius,<br>$\theta$: cylindrical angle in radians, and<br>$\zeta$: height. |
| **Domain of the generating function or mapping equations** | $\left\{(\rho,\theta,\zeta) \text{ in } \mathbf{R}^3 \;\middle|\; 0 < \rho,\ 0 \leq \theta < 2\pi, \text{ and } -\infty < \zeta < \infty\right\}$<br>$\cup \left\{(0,0,\zeta) \text{ in } \mathbf{R}^3 \;\middle|\; -\infty < \zeta < \infty\right\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}_C(\rho,\theta,\zeta) = (x,\ y,\ z)$,<br>where:<br>$x = \rho\cos\theta$,<br>$y = \rho\sin\theta$, and<br>$z = \zeta$. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |

<table>
<tr><th>Element</th><th>Specification</th></tr>
<tr><td>Inverse of the generating function or mapping equations</td><td>$\boldsymbol{F}_{\mathrm{C}}^{-1}(x,y,z)=(\rho,\theta,\zeta)$,<br>where:<br>$\rho=\sqrt{x^2+y^2}$,<br>$\theta=\begin{cases}\theta' & \text{if } \theta'\geq 0\\ 2\pi+\theta' & \text{if } \theta'<0,\end{cases}$<br>$\theta'=\operatorname{arctan2}(y,\ x)$, and<br>$\zeta=z$.</td></tr>
<tr><td>COM</td><td>n/a</td></tr>
<tr><td>Point distortion</td><td>n/a</td></tr>
<tr><td>Figures</td><td><br></td></tr>
<tr><td>Notes</td><td>The inverse generating function is discontinuous on the $z$-axis.</td></tr>
<tr><td>References</td><td>[EDM]</td></tr>
</table>

### 5.9.11 Lococentric cylindrical CS specification

**Table 5.17 — Lococentric cylindrical CS**

| Element | Specification |
|---|---|
| **Description** | Localized cylindrical. |
| **CS label** | LOCOCENTRIC_CYLINDRICAL |
| **CS code** | 10 |
| **Function type** | Generating function. |

| Element | Specification |
|---|---|
| **CS descriptor** | 3D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $\rho$: radius,<br>$\theta$: cylindrical angle in radians, and<br>$\zeta$: height. |
| **Domain of the generating function or mapping equations** | $\{(\rho,\theta,\zeta) \text{ in } \mathbf{R}^3 \mid 0 < \rho,\ 0 \le \theta < 2\pi, \text{ and } -\infty < \zeta < \infty\}$<br>$\cup \{(0,0,\zeta) \text{ in } \mathbf{R}^3 \mid -\infty < \zeta < \infty\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\rho,\theta,\zeta) = \boldsymbol{L}_{3\mathrm{D}} \circ \boldsymbol{F}_{\mathrm{C}}(\rho,\theta,\zeta)$,<br>where:<br>$\boldsymbol{L}_{3\mathrm{D}}$ = the 3D localization operator, and<br>$\boldsymbol{F}_{\mathrm{C}}$ = the cylindrical CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^3$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = \boldsymbol{F}_{\mathrm{C}}^{-1} \circ \boldsymbol{L}_{3\mathrm{D}}^{-1}(x,y,z)$,<br>where:<br>$\boldsymbol{L}_{3\mathrm{D}}^{-1}$ = the 3D localization inverse operator, and<br>$\boldsymbol{F}_{\mathrm{C}}^{-1}$ = the cylindrical CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** | See Table 5.16. |
| **Notes** | 1) The generating function is the composition of the cylindrical generating function (see Table 5.16) with the 3D localization operator (see 5.7).<br>2) The inverse generating function is discontinuous on the $z$-axis. |
| **References** | [EDM] |

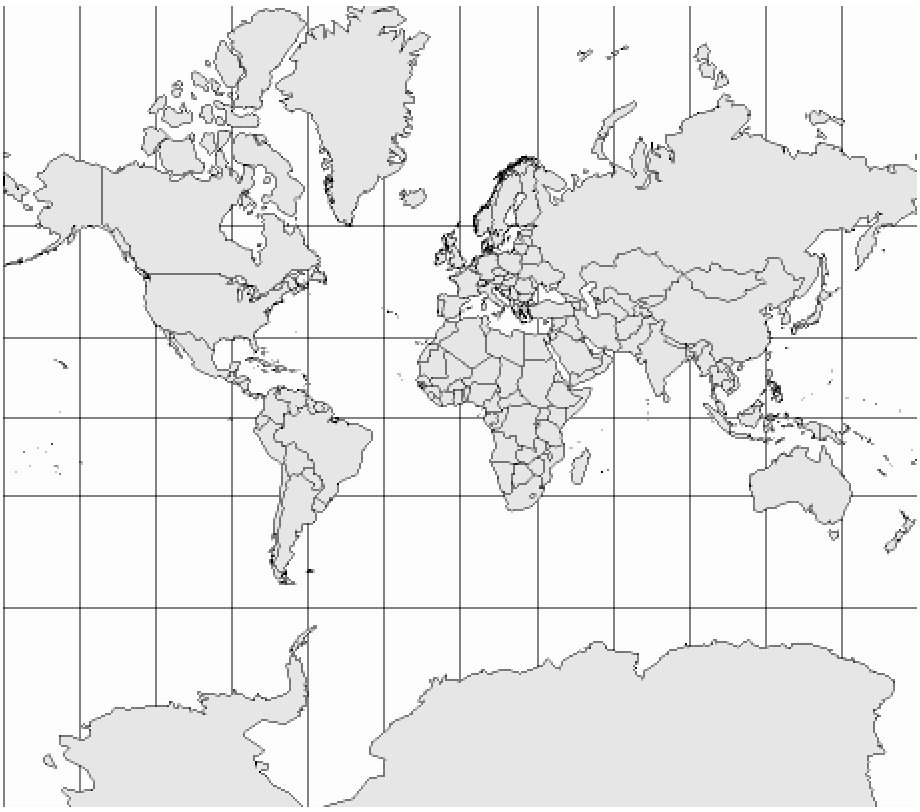
### 5.9.12 Mercator CS specification

**Table 5.18 — Mercator CS**

| Element | Specification |
|---|---|
| **Description** | Mercator and augmented Mercator map projection coordinate systems. |

| Element | Specification |
|---|---|
| **CS label** | MERCATOR |
| **CS code** | 11 |
| **Function type** | Mapping equations. |
| **CS descriptor** | Surface (map projection) and 3D (augmented map projection). |
| **Properties** | Orthogonal, conformal. |
| **CS parameters and constraints** | $a$ : oblate ellipsoid semi-major axis $(a > 0)$<br>$\varepsilon$ : oblate ellipsoid eccentricity $(\varepsilon \geq 0)$<br>$\lambda_{\text{origin}}$: longitude of origin in radians $(-\pi < \lambda_{\text{origin}} \leq \pi)$<br>$k_0$ : central scale $(0 < k_0 \leq 1)$<br>$u_{\text{F}}$: false easting<br>$v_{\text{F}}$: false northing |
| **Coordinate-components** | $u$: easting, and<br>$v$: northing.<br>Augmented coordinate:<br>$h$: ellipsoidal height |
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \leq \pi$<br>$-\pi/2 < \varphi < \pi/2$ |
| **Generating function or mapping equations** | $u = P_1(\lambda, \varphi) = u_{\text{F}} + ak_0\Lambda^*$, and<br>$v = P_2(\lambda, \varphi) = v_{\text{F}} + ak_0 \ln\left(\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1-\varepsilon\sin(\varphi)}{1+\varepsilon\sin(\varphi)}\right)^{\frac{\varepsilon}{2}}\right)$,<br>where:<br>$\Lambda^* = \Lambda_{\text{C}}(\lambda, \lambda_{\text{origin}})$. |
| **Domain of the inverse of the generating function or mapping equations** | $-\pi ak_0 < u - u_{\text{F}} \leq \pi ak_0$<br>$-\infty < v - v_{\text{F}} < \infty$ |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\lambda = Q_1(u,v) = \Lambda_C(\Lambda^*, -\lambda_{\text{origin}}),$<br>where:<br>$\Lambda^* = \dfrac{u - u_F}{ak_0}$<br>For $\varphi$, functional iteration is used for the representation of the inverse mapping equation [SNYD]. Superscripts involving $i$ indicate elements in the iteration sequence.<br>$\varphi = Q_2(u,v) = \lim\limits_{i \to \infty} Q_2^i(u,v),$<br>where:<br>$Q_2^1(u,v) = \dfrac{\pi}{2} - 2\arctan\left(\exp\left(\dfrac{-v + v_F}{ak_0}\right)\right)$, and<br>$Q_2^{i+1}(u,v) = \dfrac{\pi}{2} - 2\arctan\left\{\exp\left(\dfrac{-v + v_F}{ak_0}\right)\left(\dfrac{1 - \varepsilon\sin\left(Q_2^i(u,v)\right)}{1 + \varepsilon\sin\left(Q_2^i(u,v)\right)}\right)^{\frac{\varepsilon}{2}}\right\}$<br>for $i = 1, 2, 3, \ldots$ |
| **COM** | $\gamma(\lambda, \varphi) = 0$ |
| **Point distortion** | $k(\lambda, \varphi) = \dfrac{ak_0}{R_N(\varphi)\cos(\varphi)}$ |
| **Figures** | <br>Example Mercator map projection |

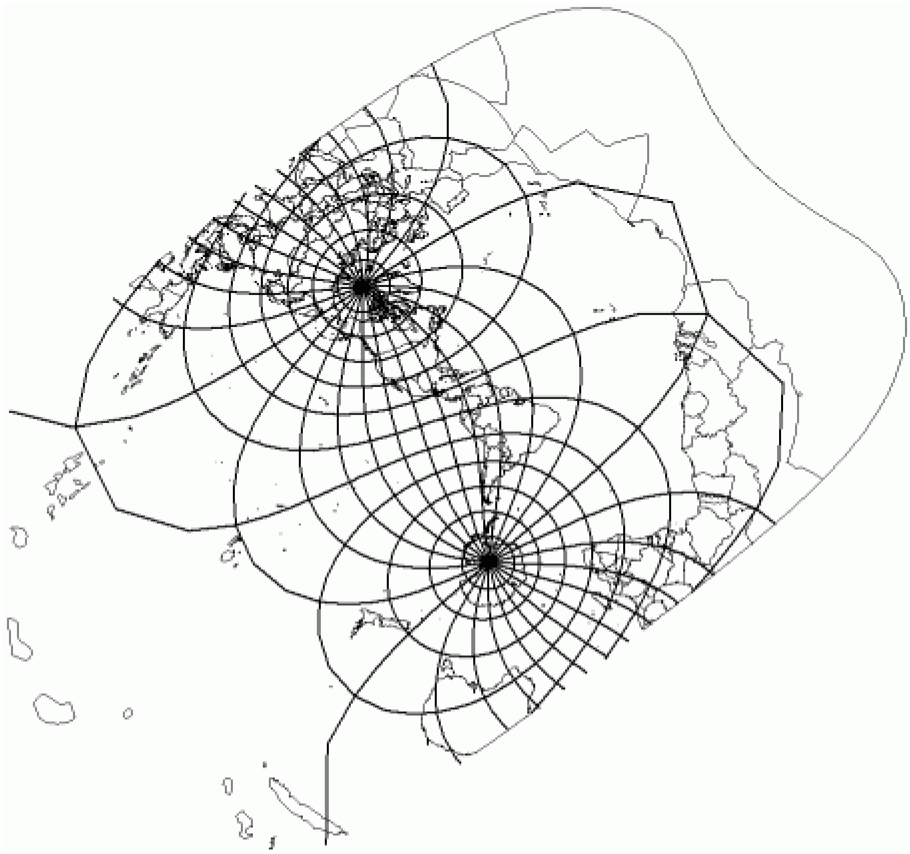
| Element | Specification |
|---|---|
| **Notes** | 1) Meridians project as straight lines that satisfy equations of the form $u$ = some constant. Equally-spaced meridians project to evenly-spaced straight lines orthogonal to the $u$-axis. Parallels project to straight lines orthogonal to the projected meridians and satisfy equations of the form $v$ = some constant. Evenly-spaced parallels project to unevenly-spaced parallel lines on the projection. The spacing of these lines increases with distance from the $u$-axis.<br>2) The meridian at $\lambda_{\text{origin}}$ corresponds to the line $u = u_{\text{F}}$.<br>3) The point distortion equals $k_0$ along the Equator.<br>4) An alternate CS parameter set is given by:<br>$a, \varepsilon, \lambda_{\text{origin}}, u_{\text{F}}, v_{\text{F}}$, and $\varphi_1$ : the secant latitude in radians $(-\pi/2 < \varphi_1 < \pi/2)$<br>This reduces to the specified CS parameter set by assigning:<br>$k_0 = \frac{1}{a} R_{\text{N}}(\varphi_1)\cos(\varphi_1)$.<br>With this value for $k_0$, $k(\lambda, \varphi_1) = 1$. |
| **References** | [SNYD] |

### 5.9.13 Oblique Mercator spherical CS specification

**Table 5.19 — Oblique Mercator spherical CS**

| Element | Specification |
|---|---|
| **Description** | Oblique Mercator and augmented oblique Mercator map projections of a sphere. |
| **CS label** | OBLIQUE_MERCATOR_SPHERICAL |
| **CS code** | 12 |
| **Function type** | Mapping equations. |
| **CS descriptor** | Surface (map projection) and 3D (augmented map projection). |
| **Properties** | Orthogonal, conformal. |
| **CS parameters and constraints** | $R$: radius of the sphere $(0 < R)$<br>$k_0$: central scale $(0 < k_0 \leq 1)$<br>$(\lambda_1, \varphi_1)$: first point on the central line<br>$(\lambda_2, \varphi_2)$: second point on central line<br>$u_{\text{F}}$: false easting<br>$v_{\text{F}}$: false northing<br>where:<br>$-\pi/2 < \varphi_1 < \pi/2,\ -\pi/2 < \varphi_2 < \pi/2,\ \lvert\varphi_1\rvert + \lvert\varphi_2\rvert > 0$, and<br>$-\pi < \lambda_1 \leq \pi,\ -\pi < \lambda_2 \leq \pi,\ \lambda_1 \neq \lambda_2,\ \lvert\lambda_1 - \lambda_2\rvert \neq \pi$. |

| Element | Specification |
|---|---|
| **Coordinate-components** | $u$: easting, and<br>$v$: northing.<br>Augmented coordinate:<br>$h$: ellipsoidal height |
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \le \pi$<br>$-\pi/2 \le \varphi \le \pi/2$<br>except for the transformed pole points determined by the values $\lambda_{\text{origin}}$ and $\alpha_0$. These values are computed in the forward mapping equations.<br>If $\alpha_0 > 0$, the transformed poles are:<br>$\left(\Lambda_C\left(-\pi/2, -\lambda_{\text{origin}}\right), \pi/2 - \alpha_0\right)$ northern hemisphere transformed pole, and<br>$\left(\Lambda_C\left(\pi/2, -\lambda_{\text{origin}}\right), -\pi/2 + \alpha_0\right)$ southern hemisphere transformed pole.<br>If $\alpha_0 < 0$, the transformed poles are:<br>$\left(\Lambda_C\left(\pi/2, -\lambda_{\text{origin}}\right), \pi/2 + \alpha_0\right)$ northern hemisphere transformed pole, and<br>$\left(\Lambda_C\left(-\pi/2, -\lambda_{\text{origin}}\right), -\pi/2 - \alpha_0\right)$ southern hemisphere transformed pole. |
| **Generating function or mapping equations** | $u = P_1(\lambda,\varphi) = u_F + Rk_0 \operatorname{arctan2}\left(p_1(\lambda,\varphi), \cos(\varphi)\cos\left(\lambda - \lambda_{\text{origin}}\right)\right),$<br>$v = P_2(\lambda,\varphi) = v_F + \frac{1}{2} Rk_0 \ln\left(\frac{1 - p_2(\lambda,\varphi)}{1 + p_2(\lambda,\varphi)}\right),$<br>where:<br>$p_1(\lambda,\varphi) = \sin(\alpha_0)\sin(\varphi) + \cos(\alpha_0)\cos(\varphi)\sin\left(\lambda - \lambda_{\text{origin}}\right),$<br>$p_2(\lambda,\varphi) = -\cos(\alpha_0)\sin(\varphi) + \sin(\alpha_0)\cos(\varphi)\sin\left(\lambda - \lambda_{\text{origin}}\right),$<br>$\alpha_0 = \begin{cases} \arctan\left(\dfrac{\sin(\varphi_1)}{\cos(\varphi_1)\sin\left(\lambda_1 - \lambda_{\text{origin}}\right)}\right) & \text{if} \left\lvert\sin\left(\lambda_1 - \lambda_{\text{origin}}\right)\right\rvert \ge \left\lvert\sin\left(\lambda_2 - \lambda_{\text{origin}}\right)\right\rvert \\ \arctan\left(\dfrac{\sin(\varphi_2)}{\cos(\varphi_2)\sin\left(\lambda_2 - \lambda_{\text{origin}}\right)}\right) & \text{if} \left\lvert\sin\left(\lambda_1 - \lambda_{\text{origin}}\right)\right\rvert < \left\lvert\sin\left(\lambda_2 - \lambda_{\text{origin}}\right)\right\rvert \end{cases},$<br>$\alpha_0$ is the principal value of the arctangent,<br>$\lambda_{\text{origin}} = \operatorname{arctan2}(p_0, q_0),$<br>$p_0 = \cos(\varphi_1)\sin(\varphi_2)\sin(\lambda_1) - \sin(\varphi_1)\cos(\varphi_2)\sin(\lambda_2)$, and<br>$q_0 = \cos(\varphi_1)\sin(\varphi_2)\cos(\lambda_1) - \sin(\varphi_1)\cos(\varphi_2)\cos(\lambda_2).$<br>Note: reversing the points $(\lambda_1,\varphi_1)$ and $(\lambda_2,\varphi_2)$ will result in the antipodal $\lambda_{\text{origin}}$. |
| **Domain of the inverse of the generating function or mapping equations** | $-\pi Rk_0 < u - u_F < \pi Rk_0$<br>$-\infty < v - v_F < \infty$ |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\lambda = Q_1(u,v) = \Lambda_C(\Lambda^*, -\lambda_{\text{origin}})$<br>$\varphi = Q_2(u,v) = \arcsin\left(\dfrac{q_2(u^*,v^*)}{\cosh(v^*)}\right)$ (principal value)<br>where:<br>$\Lambda^* = \operatorname{arctan2}\left(q_1(u^*,v^*), \cos(u^*)\right)$<br>$q_1(u^*,v^*) = \cos(\alpha_0)\sin(u^*) - \sin(\alpha_0)\sinh(v^*)$,<br>$q_2(u^*,v^*) = \sin(\alpha_0)\sin(u^*) + \cos(\alpha_0)\sinh(v^*)$,<br>$u^* = \dfrac{u - u_F}{Rk_0}$, and<br>$v^* = \dfrac{v - v_F}{Rk_0}$. |
| **COM** | $\gamma(\lambda,\varphi)$<br>$= \operatorname{arctan2}\left(-\sin(\alpha_0)\cos(\lambda - \lambda_{\text{origin}}), \cos(\alpha_0)\cos(\varphi) + \sin(\alpha_0)\sin(\varphi)\sin(\lambda - \lambda_{\text{origin}})\right)$ |
| **Point distortion** | $k(\lambda,\varphi) = \dfrac{k_0}{\sqrt{1 - \left(\cos(\alpha_0)\sin(\varphi) - \sin(\alpha_0)\cos(\varphi)\sin(\lambda - \lambda_{\text{origin}})\right)^2}}$ |
| **Figures** | <br>Example oblique Mercator spherical map projection |

| Element | Specification |
|---|---|
| Notes | 1) The method for specifying the central line by specifying two points on the central line can be accomplished by alternative formulations. The formulations for two such alternatives are provided below.<br><br>Alternative a)<br><br>In this alternative the user specifies $\lambda_{\text{origin}}$ the longitude of one of the two points where the central line crosses the equator and $\alpha_0$ the equator crossing angle at that point. The CS parameters are $\lambda_{\text{origin}}, \alpha_0, k_0, u_F$, and $v_F$.<br><br>The CS parameter constraints are:<br>$0 < \lvert\alpha_0\rvert < \pi/2$, and $-\pi < \lambda_{\text{origin}} \leq \pi$<br><br>Alternative b)<br><br>In this alternative, a point $(\lambda_1, \varphi_1)$ on the central line, a central line crossing angle $\alpha_1$ at the point, and $k_0$ at the point are specified. The central line crossing angle is the angle between the central line and the parallel through the given point. The positive sense of the angle is counter-clockwise from east. In this case the origin $(\lambda_{\text{origin}}, 0)$ and the equator crossing $\alpha_0$ are computed. The CS parameters are $\lambda_1, \varphi_1, \alpha_1, k_0, u_F$, and $v_F$.<br><br>The CS parameter constraints are:<br>$0 < \lvert\alpha_1\rvert < \pi/2, -\pi < \lambda_1 \leq \pi$ , $\lvert\varphi_1\rvert \neq \pi/2$<br><br>The mathematical formulation is,<br>$\Lambda_1^* = \text{arctan2}\left(\cos(\alpha_1)\sin(\varphi_1)\ ,\ \sin(\alpha_1)\right)$,<br>$\lambda_{\text{origin}} = \Lambda_C\left(\lambda_1, \Lambda_1^*\right)$, and<br>$\alpha_0 = \arccos\left(\cos(\alpha_1)\cos(\varphi_1)\right)$ (principal value).<br><br>2) Point distortion is equal to $k_0$ at all points on the central line. |
| References | [SNYD] |

### 5.9.14 Transverse Mercator CS specification

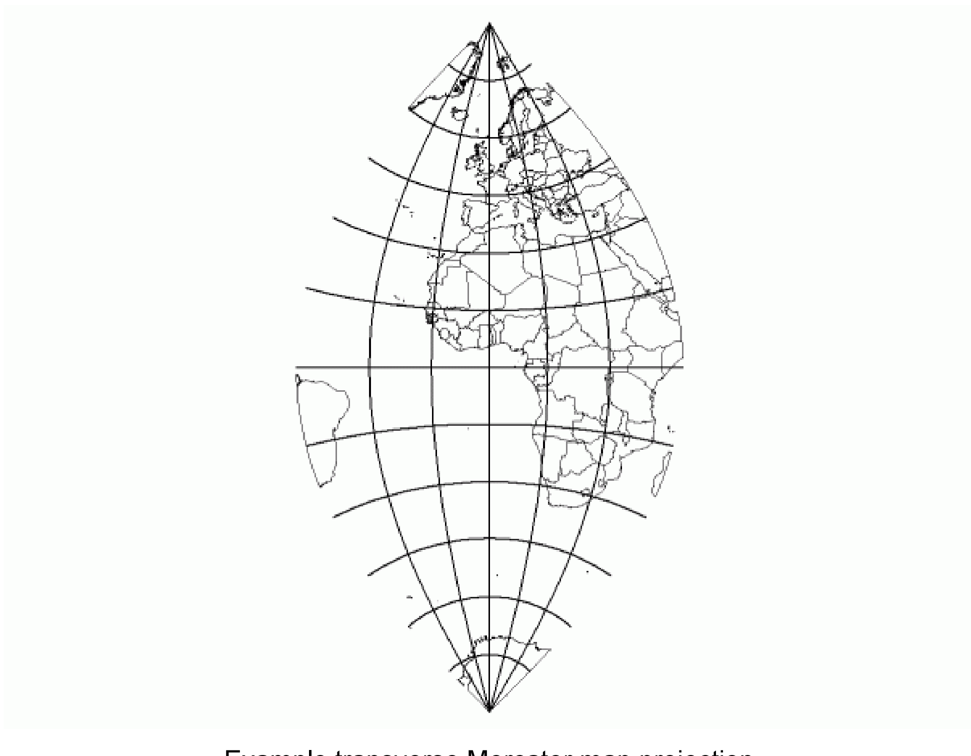
**Table 5.20 — Transverse Mercator CS**

| Element | Specification |
|---|---|
| Description | Transverse Mercator and augmented transverse Mercator map projections. |
| CS label | TRANSVERSE_MERCATOR |
| CS code | 13 |
| Function type | Mapping equations. |
| CS descriptor | Surface (map projection) and 3D (augmented map projection). |
| Properties | Orthogonal, conformal. |

| Element | Specification |
|---|---|
| **CS parameters and constraints** | $a$ : oblate ellipsoid major semi-axis $(a > 0)$<br>$\varepsilon$ : oblate ellipsoid eccentricity $(\varepsilon \geq 0)$<br>$\lambda_{\text{origin}}$: longitude of origin in radians $(-\pi < \lambda_{\text{origin}} \leq \pi)$<br>$\varphi_{\text{origin}}$: latitude of origin in radians $(-\pi/2 < \varphi_{\text{origin}} < \pi/2)$<br>$k_0$ : central scale $(0 < k_0 \leq 1)$<br>$u_{\text{F}}$: false easting<br>$v_{\text{F}}$: false northing |
| **Coordinate-components** | $u$: easting, and<br>$v$: northing.<br>Augmented coordinate:<br>$h$: ellipsoidal height |
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \leq \pi$<br>$-\pi/2 < \varphi < \pi/2$<br>Except $(\lambda, \varphi)$ satisfying the following condition:<br>$\varphi = 0$ and $\frac{\pi}{2}(1-\varepsilon) \leq \left\lvert \Lambda_{\text{C}}\left(\lambda, \lambda_{\text{origin}}\right)\right\rvert \leq \frac{\pi}{2}(1+\varepsilon)$ |
| **Generating function or mapping equations** | $u = P_1(\lambda, \varphi) = u_{\text{F}} + a k_{\text{o}} u^*$, and<br>$v = P_2(\lambda, \varphi) = v_{\text{F}} + k_{\text{o}}\left(a v^* - S\left(\varphi_{\text{origin}}\right)\right)$,<br>where:<br>$z^* = v^* + i u^*$<br>$z^* = \left(1-\varepsilon^2\right)\int_0^w \frac{dt}{\left(\text{dn}\left(t \mid \varepsilon^2\right)\right)^2}$,<br>$w$ is the solution to the equation<br>$f(w) = \zeta$<br>which is determined by Newton's method for complex functions,<br>$w = \lim_{m \to \infty} w^m$,<br>$w^0 = \arcsin\left(\tanh(\zeta)\right)$ (principal value),<br>$w^{m+1} = w^m - \frac{f\left(w^m\right) - \zeta}{f'\left(w^m\right)}, \quad m = 0, 1, 2, 3 \ldots$,<br>$f(w) = \text{arctanh}\left(\text{sn}\left(w \mid \varepsilon^2\right)\right) - \varepsilon \cdot \text{arctanh}\left(\varepsilon \cdot \text{sn}\left(w \mid \varepsilon^2\right)\right)$,<br>$f'(w) = \frac{1-\varepsilon^2}{\text{cn}\left(w \mid \varepsilon^2\right) \cdot \text{dn}\left(w \mid \varepsilon^2\right)}$,<br>$\zeta = \psi + i\Lambda^*, \quad i^2 = -1$,<br>$\psi = \text{arctanh}\left(\sin(\varphi)\right) - \varepsilon\, \text{arctanh}\left(\varepsilon \cdot \sin(\varphi)\right)$, and<br>$\Lambda^* = \Lambda_{\text{C}}\left(\lambda, \lambda_{\text{origin}}\right)$. |

| Element | Specification |
|---|---|
| **Domain of the inverse of the generating function or mapping equations** | $-k_0 a\left(1-\varepsilon^2\right)L < u - u_{\mathrm{F}} < k_0 a\left(1-\varepsilon^2\right)L$<br>$-2k_0 S\left(\pi/2\right) < v - v_{\mathrm{F}} < 2k_0 S\left(\pi/2\right)$<br>where:<br>$L = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\xi\right)}{\sqrt{1-\left(1-\varepsilon^2\right)\sin^2\left(\xi\right)}} d\xi.$ |
| **Inverse of the generating function or mapping equations** | $\lambda = Q_1\left(u,v\right) = \Lambda_{\mathrm{C}}\left(\Lambda^*, -\lambda_{\mathrm{origin}}\right),$<br>where:<br>$\zeta = \psi + i\Lambda^*, \quad i^2 = -1,$<br>$\zeta = \operatorname{arctanh}\left(\operatorname{sn}\left(w \mid \varepsilon^2\right)\right) - \varepsilon \cdot \operatorname{arctanh}\left(\varepsilon \cdot \operatorname{sn}\left(w \mid \varepsilon^2\right)\right),$<br>$w$ is determined by Newton's method for complex functions,<br>$w = \lim_{m\to\infty} w^m,$<br>$w^0 = z^*,$<br>$w^{m+1} = w^m - \frac{f\left(w^m\right) - z^*}{f'\left(w^m\right)}, \quad m = 0,1,2,3,\ldots,$<br>$f\left(w\right) = \left(1-\varepsilon^2\right)\int_0^w \frac{d\xi}{\left[\operatorname{dn}\left(\xi \mid \varepsilon^2\right)\right]^2},$<br>$f'\left(w\right) = \frac{\left(1-\varepsilon^2\right)}{\left[\operatorname{dn}\left(w \mid \varepsilon^2\right)\right]^2},$<br>$z^* = v^* + iu^*,$<br>$u^* = \frac{u - u_F}{ak_0},$<br>$v^* = \frac{v - v_{\mathrm{F}} + k_0 S\left(\varphi_{\mathrm{origin}}\right)}{ak_0}.$<br><br>$\varphi = Q_2\left(u,v\right)$ is determined by functional iteration on the equation for the isometric latitude,<br>$\varphi = \lim_{m\to\infty} \varphi^m,$<br>$\varphi^0 = 2\arctan\left(\exp\left(\psi\right)\right) - \frac{\pi}{2},$ and<br>$\varphi^{m+1} = 2\arctan\left[\left(\exp\left(\psi\right)\right)\left(\frac{1+\varepsilon\sin\left(\varphi^m\right)}{1-\varepsilon\sin\left(\varphi^m\right)}\right)^{\frac{\varepsilon}{2}}\right] - \frac{\pi}{2}, \quad m = 0,1,2,3\ldots.$ |

| Element | Specification |
|---|---|
| **COM** | $\gamma(\lambda,\varphi) = \text{arctan2}\left(\dfrac{\partial v^*}{\partial \lambda}, \dfrac{\partial u^*}{\partial \lambda}\right)$<br>where:<br>$\dfrac{\partial v^*}{\partial \lambda} + i\dfrac{\partial u^*}{\partial \lambda} = i\dfrac{\text{cn}(w \mid \varepsilon^2)}{\text{dn}(w \mid \varepsilon^2)}$<br>and $w$: the intermediate value computed in the mapping equations for $(\lambda,\varphi)$. |
| **Point distortion** | $k(\lambda,\varphi) = \dfrac{a k_{\text{o}}}{R_{\text{N}}(\varphi)\cos(\varphi)}\left\lvert\dfrac{\text{cn}(w \mid \varepsilon^2)}{\text{dn}(w \mid \varepsilon^2)}\right\rvert$<br>where $w$: the intermediate value computed in the mapping equations for $(\lambda,\varphi)$.<br>Simplification in the case of a sphere:<br>$k(\lambda,\varphi) = \dfrac{k_o}{\sqrt{1-\cos^2(\varphi)\sin^2(\lambda-\lambda_{\text{origin}})}}$. |
| **Figures** | <br>Example transverse Mercator map projection |

| Element | Specification |
|---|---|
| Notes | 1) As noted in [SNYD] and [LLEE], an iterative exact solution for the transverse Mercator forward and inverse conversions were developed by Prof. E. H. Thompson in 1945 and were formally published by L. P. Lee in 1962 [LLEE] with the permission of Professor Thompson. In 1980, J. Dozier published a report that adapted Lee's results to a computerized solution, including complete program listings in the C language [DOZI]. The formulation used here is further adapted by C. Rollins of the United States National Geospatial-Intelligence Agency (NGA) to include a central scale factor, a non-zero latitude origin and false easting and false northing offsets for both the easting and northing coordinate-components.<br>2) The complex functions sn$(w\,\vert\varepsilon^2)$, cn$(w\,\vert\varepsilon^2)$ and dn$(w\,\vert\varepsilon^2)$ are defined in A.8.2.<br>3) The CS Generation function is extended by continuity to include the oblate ellipsoid pole points.<br>4) The domain of the inverse mapping equations covers the main region of interest. This domain can be extended to a larger region whose exact specification is complicated to define.<br>5) The iterative procedures used in both the forward and inverse formulations may be numerically ill conditioned near certain points. This occurs near the boundaries of the domains involved and near the equator for points with negative $u$ values. When implementing these methods in software, special numerical methods may be required in the neighbourhood of such exceptional points. In particular, in the forward conversion, the exceptional points on the equator are avoided by restricting the procedure to use $\vert\Lambda^*\vert$ and then setting $u$ to be negative when $\Lambda^*<0$.<br>6) The point distortion equals $k_0$ along the meridian at $\lambda_{\text{origin}}$. |
| References | [DOZI], [LLEE], and [SNYD]. |

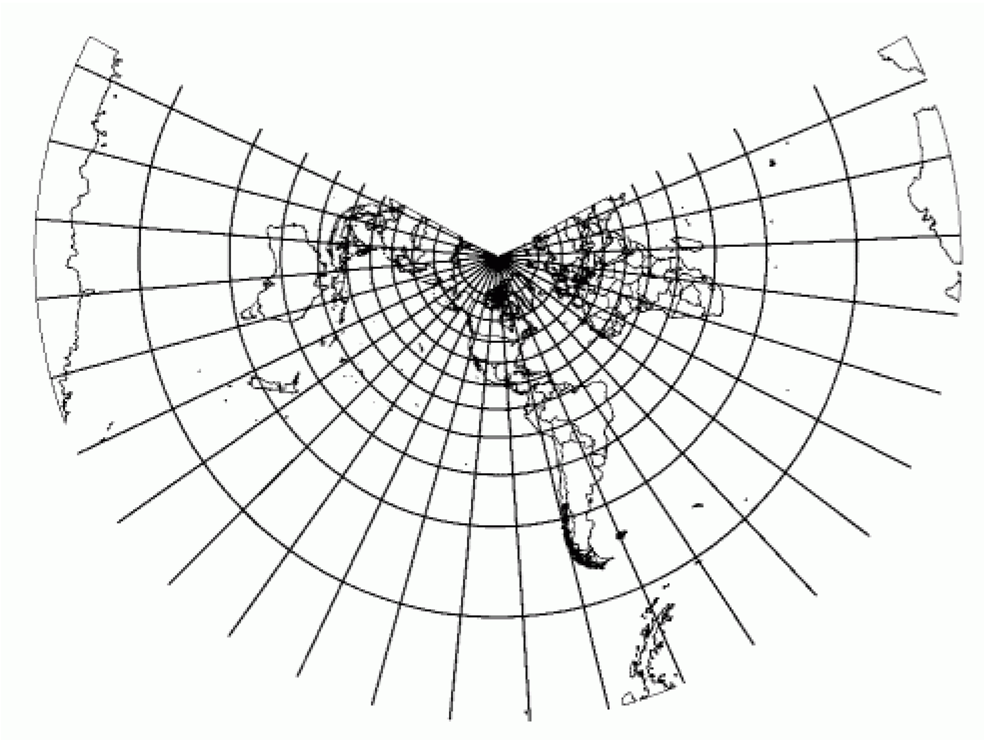
### 5.9.15 Lambert conformal conic CS specification

**Table 5.21 — Lambert conformal conic CS**

| Element | Specification |
|---|---|
| Description | Lambert conformal conic and augmented Lambert conformal conic map projections. |
| CS label | LAMBERT_CONFORMAL_CONIC |
| CS code | 14 |
| Function type | Mapping equations. |
| CS descriptor | Surface (map projection) and 3D (augmented map projection). |
| Properties | Orthogonal, conformal. |

| Element | Specification |
|---|---|
| **CS parameters and constraints** | $a$: oblate ellipsoid major semi-axis ($a > 0$)<br>$\varepsilon$: oblate ellipsoid eccentricity ($\varepsilon \geq 0$)<br>$\varphi_{\text{origin}}$: latitude of the origin in radians ( $-\pi/2 < \varphi_{\text{origin}} < \pi/2$)<br>$\lambda_{\text{origin}}$: longitude of origin in radians ( $-\pi < \lambda_{\text{origin}} \leq \pi$)<br>$\varphi_1, \varphi_2$: secant latitudes in radians ($-\pi/2 < \varphi_1 < \pi/2,\ -\pi/2 < \varphi_2 < \pi/2$)<br>$\varphi_1, \varphi_2$: $\varphi_1 \neq -\varphi_2$<br>$u_{\text{F}}$: false easting<br>$v_{\text{F}}$: false northing |
| **Coordinate-components** | $u$: easting, and<br>$v$: northing.<br>Augmented coordinate:<br>$h$: ellipsoidal height |
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \leq \pi$<br>$-\pi/2 < \varphi < \pi/2$ |
| **Generating function or mapping equations** | $u = P_1(\lambda, \varphi) = u_{\text{F}} + \rho(\varphi)\sin(n\Lambda^*)$, and<br>$v = P_2(\lambda, \varphi) = v_{\text{F}} + \rho(\varphi_{\text{origin}}) - \rho(\varphi)\cos(n\Lambda^*)$,<br>where:<br>$\Lambda^* = \Lambda_{\text{C}}(\lambda, \lambda_{\text{origin}})$,<br>$\rho(\varphi) = \rho_0 \left( \dfrac{\tau(\varphi)}{\tau(\varphi_0)} \right)^n$,<br>$\rho_0 = a\,k_0\,m(\varphi_0)/n$,<br>$\tau(\varphi) = \tan\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)\left[\dfrac{1+\varepsilon\sin(\varphi)}{1-\varepsilon\sin(\varphi)}\right]^{\frac{\varepsilon}{2}}$,<br>$m(\varphi) = \dfrac{\cos(\varphi)}{\sqrt{1-\varepsilon^2\sin^2(\varphi)}}$,<br>$k_0 = \dfrac{m(\varphi_1)}{m(\varphi_0)}\left(\dfrac{\tau(\varphi_0)}{\tau(\varphi_1)}\right)^n$,<br>$\varphi_0 = \arcsin(n)$ (principal value), and<br>$n = \begin{cases} \sin(\varphi_1) & \text{if } \varphi_1 = \varphi_2 \\ \dfrac{\ln(m(\varphi_1)) - \ln(m(\varphi_2))}{\ln(\tau(\varphi_1)) - \ln(\tau(\varphi_2))} & \text{if } \varphi_1 \neq \varphi_2 \end{cases}$. |
| **Domain of the inverse of the generating function or mapping equations** | $(u, v) \neq (u_{\text{F}}, v_{\text{F}} + \rho(\varphi_{\text{origin}}))$ and $-\pi\lvert n\rvert < \text{arctan2}(u - u_{\text{F}},\ \rho(\varphi_{\text{origin}}) - v + v_{\text{F}}) < \pi\lvert n\rvert$ |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\lambda = Q_1(u,v) = \Lambda_C(\Lambda^*, -\lambda_{origin})$, and<br>$\varphi = Q_2(u,v) = \lim_{m \to \infty} Q_2^m(u,v)$,<br>where:<br>$\Lambda^* = \frac{1}{n}\text{arctan2}\left(\text{sgn}(n)(u-u_F),\ \text{sgn}(n)\left(\rho(\varphi_{origin}) - v + v_F\right)\right)$,<br>$Q_2^0(u,v) = \frac{\pi}{2} - 2\arctan\left(t(u,v)\right)$,<br>$Q_2^{m+1}(u,v) = \frac{\pi}{2} - 2\arctan\left\{t(u,v)\left(\frac{1-\varepsilon\sin\left(Q_2^m(u,v)\right)}{1+\varepsilon\sin\left(Q_2^m(u,v)\right)}\right)^{\frac{\varepsilon}{2}}\right\}$, for $m = 1,2,3,\ldots$,<br>$t(u,v) = \tau(\varphi_0)\left(r(u,v)/\rho_0\right)^{1/n}$,<br>$r(u,v) = \text{sgn}(n)\sqrt{(u-u_F)^2 + \left[\rho(\varphi_{origin}) - v + v_F\right]^2}$, and<br>$n, \rho_0, \tau(\varphi_0)$ and $\rho(\varphi_{origin})$ are defined in the forward mapping equations field. |
| **COM** | $\gamma(\lambda,\varphi) = n\Lambda^*$ where: $\Lambda^* = \Lambda_C(\lambda, \lambda_{origin})$. |
| **Point distortion** | $k(\lambda,\varphi) = \frac{m(\varphi_1)}{m(\varphi)}\left(\frac{\tau(\varphi)}{\tau(\varphi_1)}\right)^n$<br>$= \frac{n\,\rho(\varphi)}{a\,m(\varphi)}$ (equivalent expression) |

| Element | Specification |
|---|---|
| Figures | <br>Example Lambert conformal conic map projection |
| Notes | 1) $\mathrm{sgn}(x)$ is the signum function (see ISO 80000-2).<br>2) The surface geodetic coordinate $(\lambda_{\text{origin}}, \varphi_{\text{origin}})$ projects to map coordinate $(u_{\text{F}}, v_{\text{F}})$.<br>3) The point distortion is unity along the standard parallel(s) $\varphi_1$ and $\varphi_2$.<br>4) An alternate CS parameter set is given by:<br>$a, \varepsilon, \varphi_{\text{origin}}, \lambda_{\text{origin}}, u_{\text{F}}, v_{\text{F}}$, and central scale $k_0$, $(0 < k_0 \le 1)$<br>where $k_0$ is a user specified point distortion at $(\lambda_{\text{origin}}, \varphi_{\text{origin}})$.<br>In this case, the $k_0$ replaces the computed intermediate variable in the mapping equations and the computed intermediate variables $\varphi_0$ and $n$ are set as $\varphi_0 = \varphi_{\text{origin}}$, and $n = \sin(\varphi_0)$. |
| References | [SNYD] |

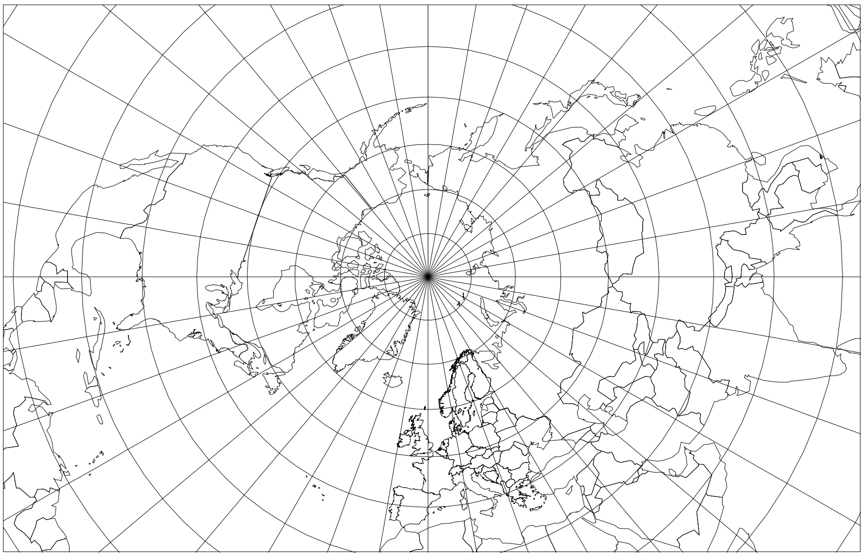
### 5.9.16 Polar stereographic CS specification

**Table 5.22 — Polar stereographic CS**

| Element | Specification |
|---|---|
| **Description** | Polar stereographic and augmented polar stereographic map projections. |
| **CS label** | POLAR_STEREOGRAPHIC |
| **CS code** | 15 |

| Element | Specification |
|---|---|
| **Function type** | Mapping equations. |
| **CS descriptor** | Surface (map projection) and 3D (augmented map projection). |
| **Properties** | Orthogonal, conformal. |
| **CS parameters and constraints** | $a$ : oblate ellipsoid major semi-axis ($a > 0$)<br>$\varepsilon$ : oblate ellipsoid eccentricity ($\varepsilon \geq 0$)<br>polar aspect: north or south<br>$\lambda_{\text{origin}}$: longitude of origin in radians ( $-\pi < \lambda_{\text{origin}} \leq \pi$)<br>$\varphi_{\text{origin}} = \begin{cases} +\pi/2 & \text{north aspect} \\ -\pi/2 & \text{south aspect} \end{cases}$<br>$k_0$ : central scale ($1/2 < k_0 \leq 1$)<br>$u_F$: false easting<br>$v_F$: false northing |
| **Coordinate-components** | $u$: easting, and<br>$v$: northing.<br>Augmented coordinate:<br>$h$: ellipsoidal height |
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \leq \pi$<br>$0 \leq \varphi < \pi/2$ and $(0, \pi/2)$ north aspect<br>$-\pi/2 < \varphi \leq 0$ and $(0, -\pi/2)$ south aspect |
| **Generating function or mapping equations** | north aspect:<br>$u = P_1(\lambda, \varphi) = u_F + \rho(\varphi)\sin(\lambda - \lambda_{\text{origin}})$,<br>$v = P_2(\lambda, \varphi) = v_F - \rho(\varphi)\cos(\lambda - \lambda_{\text{origin}})$,<br>south aspect:<br>$u = P_1(\lambda, \varphi) = u_F + \rho(\varphi)\sin(\lambda - \lambda_{\text{origin}})$,<br>$v = P_2(\lambda, \varphi) = v_F + \rho(\varphi)\cos(\lambda - \lambda_{\text{origin}})$,<br>where:<br>$\rho(\varphi) = 2a\,k_0 E\tau(\varphi)$,<br>$\tau(\varphi) = \tan\left(\frac{\pi}{4} - \frac{\lvert\varphi\rvert}{2}\right)\left[\frac{1+\varepsilon\sin(\lvert\varphi\rvert)}{1-\varepsilon\sin(\lvert\varphi\rvert)}\right]^{\frac{\varepsilon}{2}}$, and<br>$E = \frac{a}{b}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\frac{\varepsilon}{2}}$. |
| **Domain of the inverse of the generating function or mapping equations** | $(u - u_F)^2 + (v - v_F)^2 \leq (2a\,k_0 E)^2$<br>$E$ is defined in the forward mapping equations field. |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\lambda = Q_1(u,v) = \Lambda_C(\Lambda^*, -\lambda_{\text{origin}})$,<br>where:<br>$\Lambda^* = \begin{cases} \text{arctan2}(u-u_F,\ -v+v_F) & \text{north aspect} \\ \text{arctan2}(u-u_F,\ v-v_F) & \text{south aspect} \end{cases}$<br>For $\varphi$, functional iteration is used for the representation of the inverse mapping equation. Superscripts involving $m$ indicate elements in the iteration sequence.<br>$\varphi = \begin{cases} \lim_{m\to\infty} Q_2^m(u,v) & \text{north aspect} \\ -\lim_{m\to\infty} Q_2^m(u,v) & \text{south aspect} \end{cases}$<br>where:<br>$Q_2^0(u,v) = \frac{\pi}{2} - 2\arctan(t(u,v))$,<br>$Q_2^{m+1}(u,v) = \frac{\pi}{2} - 2\arctan\left\{ t(u,v)\left(\frac{1-\varepsilon\sin(Q_2^m(u,v))}{1+\varepsilon\sin(Q_2^m(u,v))}\right)^{\frac{\varepsilon}{2}} \right\}$ for $m = 1,2,3,\ldots$,<br>$t(u,v) = \frac{\sqrt{(u-u_F)^2 + (v-v_F)^2}}{2ak_0E}$.<br>$E$ is defined in the forward mapping equations field. |
| **COM** | $\gamma(\lambda,\varphi) = \begin{cases} \Lambda^* & \text{north aspect} \\ -\Lambda^* & \text{south aspect} \end{cases}$ where: $\Lambda^* = \Lambda_C(\lambda, \lambda_{\text{origin}})$. |
| **Point distortion** | $k(\lambda,\varphi) = \frac{2ak_0E\tau(\varphi)}{R_N(\varphi)\cos(\varphi)}$<br>where: $\tau(\varphi)$ and $E$ are defined in the forward mapping equations field. |
| **Figures** | <br>Example polar stereographic map projection<br>See also Figure 5.7 |

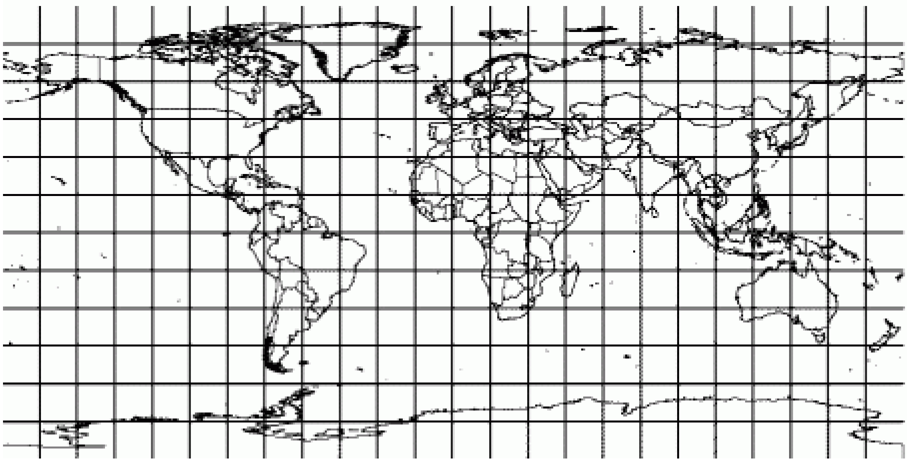
 

| Element | Specification |
|---|---|
| **Notes** | 1) Meridians project as straight lines radiating from the point $(u_F, v_F)$. Parallels project to concentric circles.<br>2) The point distortion values at pole is: $k(\lambda, \varphi_{origin}) = k_0$.<br>3) An alternate CS parameter set is given by: $a, \varepsilon, \varphi_{origin}, \lambda_{origin}, u_F, v_F$, and $\varphi_1$ : the secant latitude in radians<br>$0 \le \varphi_1 < \pi/2$ north aspect,<br>$-\pi/2 < \varphi_1 \le 0$ south aspect.<br>This reduces to the specified CS parameter set by assigning:<br>$k_0 = \dfrac{R_N(\varphi_1)\cos(\varphi_1)}{2aE\tau(\varphi_1)}$<br>where: $\tau(\varphi)$ and $E$ are defined in the forward mapping equations field.<br>With this value for $k_0$, $k(\lambda, \varphi_1) = 1$.<br>4) In the case of a sphere, the mapping equations are derived from the stereographic projection (see Figure 5.7).<br>5) The CS Generation function is extended by continuity to include the oblate ellipsoid pole points. |
| **References** | [SNYD] |

### 5.9.17 Equidistant cylindrical CS specification

**Table 5.23 — Equidistant cylindrical CS**

| Element | Specification |
|---|---|
| **Description** | Equidistant cylindrical and augmented equidistant cylindrical map projections. |
| **CS label** | EQUIDISTANT_CYLINDRICAL |
| **CS code** | 16 |
| **Function type** | Mapping equations. |
| **CS descriptor** | Surface (map projection) and 3D (augmented map projection). |
| **Properties** | Orthogonal, non-conformal. |
| **CS parameters and constraints** | $a$ : oblate ellipsoid semi-major axis ($a > 0$)<br>$\varepsilon$ : oblate ellipsoid eccentricity ($\varepsilon \ge 0$)<br>$\lambda_{origin}$: longitude of origin in radians $(-\pi < \lambda_{origin} \le \pi)$<br>$k_0$ : central scale $(0 < k_0 \le 1)$<br>$u_F$: false easting<br>$v_F$: false northing |
| **Coordinate-components** | $u$: easting, and<br>$v$: northing.<br>Augmented coordinate:<br>$h$: ellipsoidal height |

| Element | Specification |
|---|---|
| **Domain of the generating function or mapping equations** | $-\pi < \lambda \le \pi$<br>$-\pi/2 < \varphi < \pi/2$ |
| **Generating function or mapping equations** | $u = P_1(\lambda,\varphi) = u_F + ak_0\Lambda^*,$<br>$v = P_2(\lambda,\varphi) = v_F + S(\varphi),$<br>where:<br>$\Lambda^* = \Lambda_C(\lambda, \lambda_{origin}).$<br>if $\varepsilon = 0$, $v = P_2(\lambda,\varphi) = v_F + a\varphi$ |
| **Domain of the inverse of the generating function or mapping equations** | $-\pi ak_0 < u - u_F \le \pi ak_0$<br>$-S(\pi/2) < v - v_F < S(\pi/2)$ |
| **Inverse of the generating function or mapping equations** | $\lambda = Q_1(u,v) = \Lambda_C\left(\frac{u-u_F}{ak_0}, -\lambda_{origin}\right),$<br>$\varphi = Q_2(u,v) = S^{-1}(v - v_F).$<br>If $\varepsilon = 0$, $\varphi = Q_2(u,v) = \frac{v-v_F}{a}.$ |
| **COM** | $\gamma(\lambda,\varphi) = 0$ |
| **Point distortion** | $j(\lambda,\varphi) = 1$ latitudinal point distortion<br>$k(\lambda,\varphi) = \frac{ak_0}{R_N(\varphi)\cos(\varphi)}$ longitudinal point distortion |
| **Figures** | <br>Example equidistant cylindrical map projection |

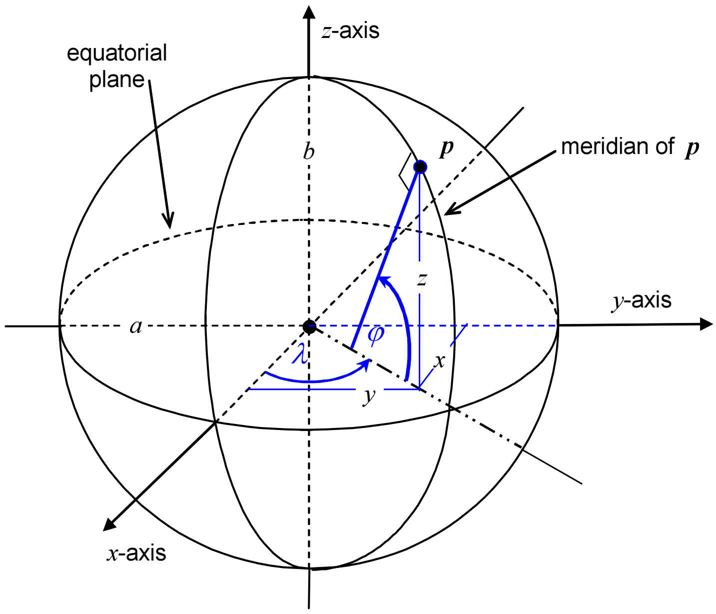
| Element | Specification |
|---|---|
| **Notes** | 1) Meridians project as straight lines that satisfy equations of the form $u$ = some constant. Equally-spaced meridians project to evenly-spaced straight lines orthogonal to the $u$-axis. Parallels project to straight lines orthogonal to the projected meridians and satisfy equations of the form $v$ = some constant.<br>2) $k(\lambda,\varphi)=k_0$ on the equator $(\varphi=0)$. $j(\lambda,\varphi)=1$ indicates true scale, or "equidistance" along meridians.<br>3) The radius $R$ of the conceptual cylinder is $R = a\,k_0$.<br>4) An alternate CS parameter set is given by:<br>$a,\varepsilon,\lambda_{\text{origin}},u_{\text{F}},v_{\text{F}}$, and $\varphi_1$ the northern secant latitiude in radians $(0\le\varphi_1<\pi/2)$.<br>This reduces to the specified CS parameter set by assigning:<br>$k_0=\frac{1}{a}R_{\text{N}}(\varphi_1)\cos(\varphi_1)$.<br>In this case, $k(\lambda,\varphi)=1$ at the secant latitiudes $\varphi=\pm\varphi_1$ |
| **References** | [SNYD] |

### 5.9.18 Surface geodetic CS specification

**Table 5.24 — Surface geodetic CS**

| Element | Specification |
|---|---|
| **Description** | Surface geodetic. |
| **CS label** | SURFACE_GEODETIC |
| **CS code** | 17 |
| **Function type** | Generating function. |
| **CS descriptor** | Surface curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | $a$: major semi-axis length<br>$b$: minor semi-axis length<br>Constraints:<br>$a > b$: (oblate ellipsoid)<br>$a = b$: (sphere) |
| **Coordinate-components** | $\lambda$ : longitude in radians, and<br>$\varphi$ : geodetic latitude in radians. |
| **Domain of the generating function or mapping equations** | $-\pi<\lambda\le\pi$<br>$-\pi/2<\varphi<\pi/2$<br>and: $(0,\pm\pi/2)$ |

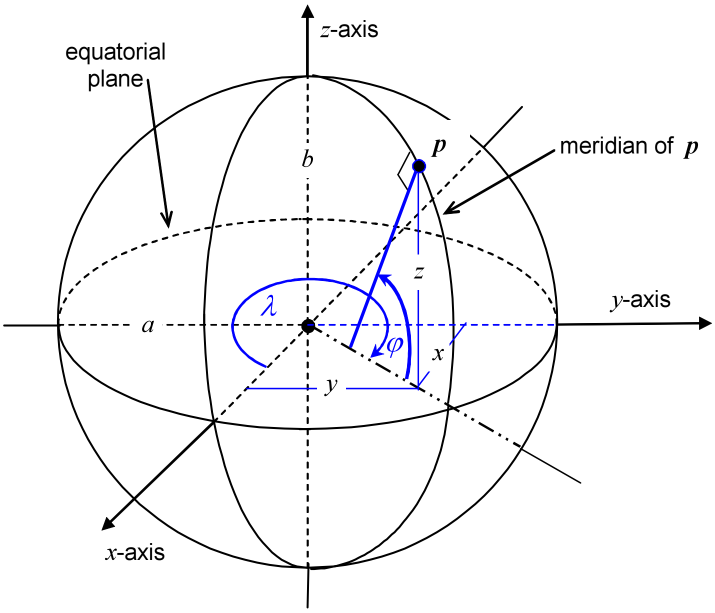
| Element | Specification |
|---|---|
| **Generating function or mapping equations** | $\boldsymbol{F}(\lambda,\varphi)=(x,y,z)$,<br>where:<br>$x = R_N(\varphi)\cos(\varphi)\cos(\lambda)$,<br>$y = R_N(\varphi)\cos(\varphi)\sin(\lambda)$, and<br>$z = (1-\varepsilon^2)R_N(\varphi)\sin(\varphi)$.<br>Simplification if $a = b = r$:<br>$\boldsymbol{F}(\lambda,\varphi)=(x,y,z)$,<br>where:<br>$x = r\cos(\varphi)\cos(\lambda)$,<br>$y = r\cos(\varphi)\sin(\lambda)$, and<br>$z = r\sin(\varphi)$. |
| **Domain of the inverse of the generating function or mapping equations** | $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}=1$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z)=(\lambda,\varphi)$,<br>where:<br>$\lambda = \arctan2(y, x)$, and<br>$\varphi = \arctan2\left(a^2 z,\, b^2\sqrt{x^2+y^2}\right)$.<br>Simplification if $a = b = r$:<br>$\varphi = \arcsin(z/r)$ (principal value). |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |

| Element | Specification |
|---|---|
| **Notes** | 1) The CS surface is the oblate ellipsoid (or sphere) surface excluding the pole points.<br>2) The geodetic 3D CS (Table 5.14) induces this CS on the 3rd coordinate-component surface at any point for which $h = 0$.<br>3) If $a = b$, the geodetic latitude $\varphi$ coincides with the spherical latitude $\theta$ (see Table 5.10).<br>4) The inverse generating function is not continuous at the pole points $(0, \pm\pi/2)$. |
| **References** | [HEIK] |

### 5.9.19 Surface planetodetic CS specification

**Table 5.25 — Surface planetodetic CS**

| Element | Specification |
|---|---|
| **Description** | Surface planetodetic. Surface geodetic with longitude in opposite direction. |
| **CS label** | SURFACE_PLANETODETIC |
| **CS code** | 18 |
| **Function type** | Generating function. |
| **CS descriptor** | Surface curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | $a$: major semi-axis length<br>$b$: minor semi-axis length<br>Constraints:<br>$a > b$: (oblate ellipsoid)<br>$a = b$: (sphere) |
| **Coordinate-components** | $\varphi$ : geodetic latitude in radians, and<br>$\lambda$ : planetodetic longitude in radians. |
| **Domain of the generating function or mapping equations** | $-\pi/2 < \varphi < \pi/2$<br>$-\pi < \lambda \leq \pi$<br>and: $(0, \pm\pi/2)$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\varphi, \lambda) = \boldsymbol{F}_{\mathrm{GD}}(-\lambda, \varphi)$,<br>where:<br>$\boldsymbol{F}_{\mathrm{GD}}$ is the surface geodetic generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$ |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z)=(\varphi,\lambda)$,<br>where:<br>$(\lambda,\varphi)=\boldsymbol{F}_{\mathrm{GD}}^{-1}(x,-y,z)$, and<br>$\boldsymbol{F}_{\mathrm{GD}}^{-1}$ is the surface geodetic inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |
| **Notes** | 1) Similar to surface geodetic CS (see Table 5.24) except that longitude is in the opposite direction. In particular, points on a planet surface rotating (prograde) into view have larger planetodetic longitudes than those points rotating out of view.<br>2) The inverse generating function is not continuous at the pole points $(0,\pm\pi/2)$.<br>3) The coordinate-components are ordered for compatibility with planetodetic 3D CS (see Table 5.15) |
| **References** | [RIIC] |

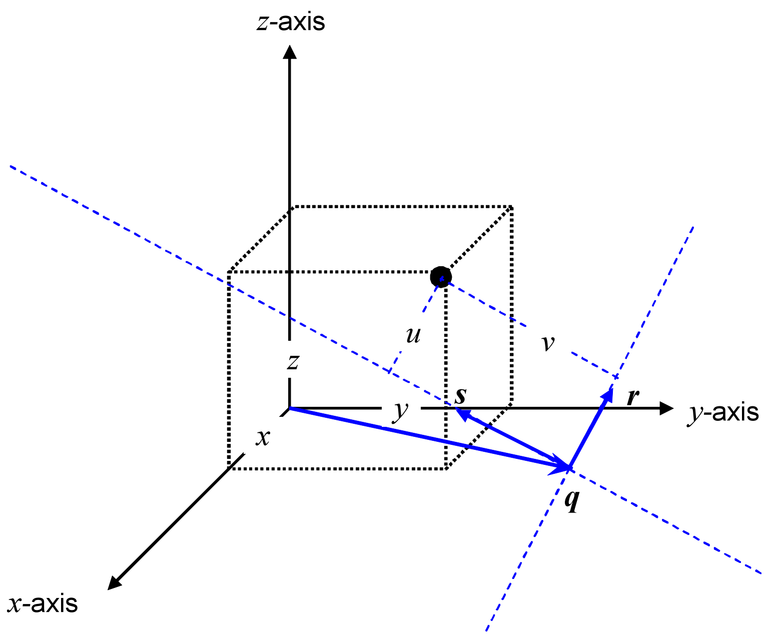
### 5.9.20 Lococentric surface Euclidean CS specification

**Table 5.26 — Lococentric surface Euclidean CS**

| Element | Specification |
|---|---|
| **Description** | Localization of Euclidean 2D CS into plane surface in 3D position-space. |
| **CS label** | LOCOCENTRIC_SURFACE_EUCLIDEAN |
| **CS code** | 19 |
| **Function type** | Generating function. |

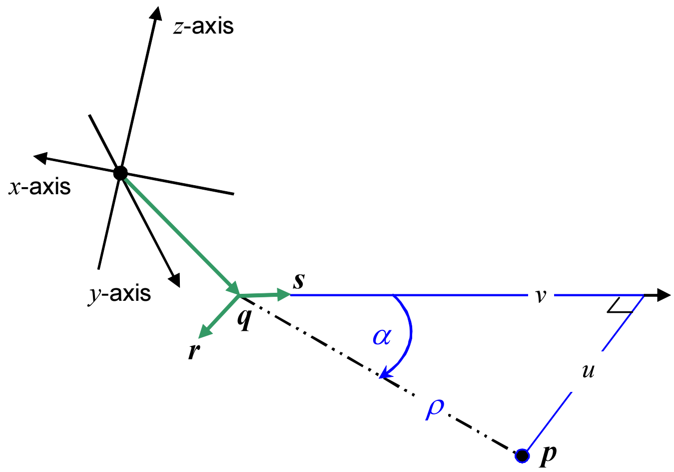
| Element | Specification |
|---|---|
| **CS descriptor** | Surface linear. |
| **Properties** | Orthonormal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $u$, $v$ |
| **Domain of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(u,v) = \boldsymbol{L}_{\text{Surface}} \circ \boldsymbol{F}_{\text{E2D}}(u,v)$<br>where:<br>$\boldsymbol{L}_{\text{Surface}}$ = the surface localization operator, and<br>$\boldsymbol{F}_{\text{E2D}}$ = the Euclidean 2D CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\left\{ \boldsymbol{p} = (x,y,z) \text{ in } \mathbf{R}^3 \middle| (\boldsymbol{p} - \boldsymbol{q}) \bullet (\boldsymbol{r} \times \boldsymbol{s}) = 0 \right\}$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = \boldsymbol{F}_{\text{E2D}}^{-1} \circ \boldsymbol{L}_{\text{Surface}}^{-1}(x,y,z)$<br>where:<br>$\boldsymbol{L}_{\text{Surface}}^{-1}$ = the surface localization inverse operator, and<br>$\boldsymbol{F}_{\text{E2D}}^{-1}$ = the Euclidean 2D CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |

| Element | Specification |
|---|---|
| Figures |  |
| Notes | 1) The CS surface is the plane specified by: $0 = f(\boldsymbol{p}) = (\boldsymbol{p} - \boldsymbol{q}) \bullet (\boldsymbol{r} \times \boldsymbol{s})$.<br>2) The generating function is the composition of the generating function for Euclidean 2D (see Table 5.29) with the surface localization operator (see 5.7). |
| References | [EDM] |

### 5.9.21 Lococentric surface azimuthal CS specification

**Table 5.27 — Lococentric surface azimuthal CS**

| Element | Specification |
|---|---|
| **Description** | Localization of azimuthal CS into plane surface in 3D position-space. |
| **CS label** | LOCOCENTRIC_SURFACE_AZIMUTHAL |
| **CS code** | 20 |
| **Function type** | Generating function. |
| **CS descriptor** | Surface curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$: the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$: axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $\alpha$: azimuth in radians, and<br>$\rho$: radius. |

| Element | Specification |
|---|---|
| **Domain of the generating function or mapping equations** | $\{(\alpha,\rho) \text{ in } \mathbf{R}^2 \mid 0 \le \alpha < 2\pi, \text{ and } 0 < \rho\} \cup \{(0,0)\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\alpha,\rho) = \boldsymbol{L}_{\text{Surface}} \circ \boldsymbol{F}_{\text{A}}(\alpha,\rho)$,<br>where:<br>$\boldsymbol{L}_{\text{Surface}}$ = the surface localization operator, and<br>$\boldsymbol{F}_{\text{A}}$ = the azimuthal CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\{\boldsymbol{p} = (x,y,z) \text{ in } \mathbf{R}^3 \mid (\boldsymbol{p}-\boldsymbol{q})\bullet(\boldsymbol{r}\times\boldsymbol{s}) = 0\}$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = \boldsymbol{F}_{\text{A}}^{-1} \circ \boldsymbol{L}_{\text{Surface}}^{-1}(x,y,z)$,<br>where:<br>$\boldsymbol{L}_{\text{Surface}}^{-1}$ = the surface localization inverse operator, and<br>$\boldsymbol{F}_{\text{A}}^{-1}$ = the azimuthal CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |
| **Notes** | 1) The CS surface is the plane specified by: $0 = f(\boldsymbol{p}) = (\boldsymbol{p}-\boldsymbol{q})\bullet(\boldsymbol{r}\times\boldsymbol{s})$.<br>2) The generating function is the composition of the generating function for azimuthal CS (see Table 5.31) with the surface localization operator (see 5.7). |
| **References** | [EDM] |

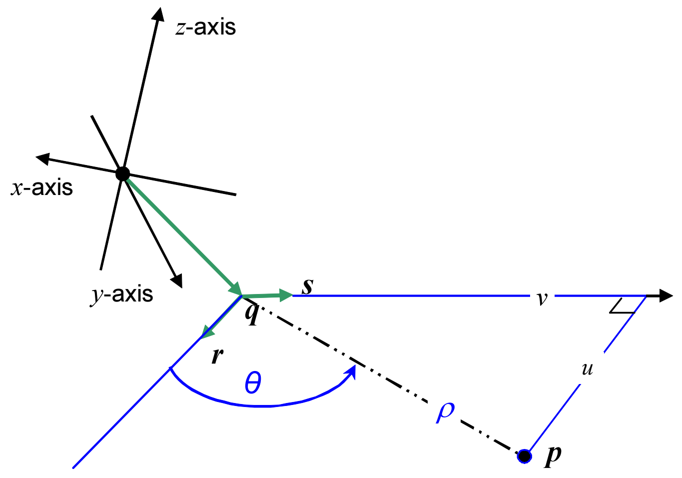
### 5.9.22 Lococentric surface polar CS specification

**Table 5.28 — Lococentric surface polar CS**

| Element | Specification |
|---|---|
| **Description** | Localization of polar CS into plane surface in 3D position-space. |

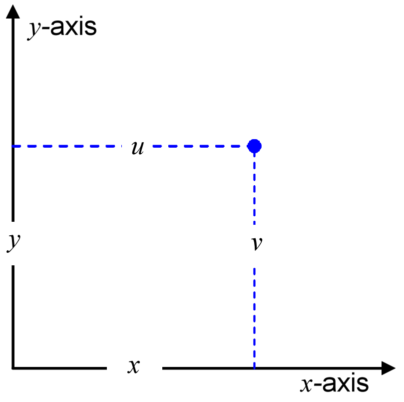
| Element | Specification |
|---|---|
| **CS label** | LOCOCENTRIC_SURFACE_POLAR |
| **CS code** | 21 |
| **Function type** | Generating function. |
| **CS descriptor** | Surface curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^3$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^3$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $\rho$: radius, and<br>$\theta$: angle in radians. |
| **Domain of the generating function or mapping equations** | $\{(\rho,\theta) \text{ in } \mathbf{R}^2 \mid 0 < \rho \text{ and } 0 \le \theta < 2\pi\} \cup \{(0,0)\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\rho,\theta) = \boldsymbol{L}_{\text{Surface}} \circ \boldsymbol{F}_{\mathbf{P}}(\rho,\theta)$,<br>where:<br>$\boldsymbol{L}_{\text{Surface}}$ = the surface localization operator, and<br>$\boldsymbol{F}_{\mathbf{P}}$ = the polar CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\{\boldsymbol{p} = (x,y,z) \text{ in } \mathbf{R}^3 \mid (\boldsymbol{p}-\boldsymbol{q}) \bullet (\boldsymbol{r} \times \boldsymbol{s}) = 0\}$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = \boldsymbol{F}_{\mathbf{P}}^{-1} \circ \boldsymbol{L}_{\text{Surface}}^{-1}(x,y,z)$,<br>where:<br>$\boldsymbol{L}_{\text{Surface}}^{-1}$ = the surface localization inverse operator, and<br>$\boldsymbol{F}_{\mathbf{P}}^{-1}$ = the polar CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |

| Element | Specification |
|---|---|
| **Figures** |  |
| **Notes** | 1) The CS surface is the plane specified by: $0 = f(\boldsymbol{p}) = (\boldsymbol{p} - \boldsymbol{q}) \bullet (\boldsymbol{r} \times \boldsymbol{s})$.<br>2) The generating function is the composition of the generating function for polar CS (see Table 5.33) with the surface localization operator (see 5.7). |
| **References** | [EDM] |

### 5.9.23 Euclidean 2D CS specification

**Table 5.29 — Euclidean 2D CS**

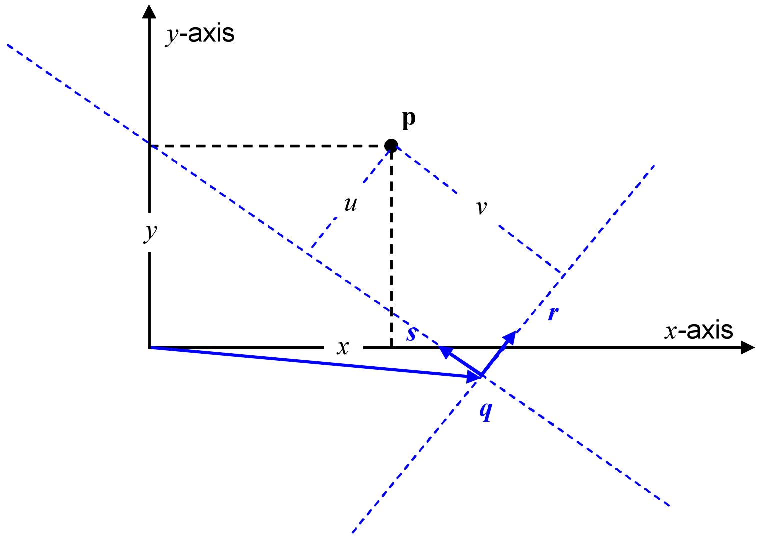
| Element | Specification |
|---|---|
| **Description** | Euclidean 2D. |
| **CS label** | EUCLIDEAN_2D |
| **CS code** | 22 |
| **Function type** | Generating function. |
| **CS descriptor** | 2D linear. |
| **Properties** | Orthonormal. |
| **CS parameters and constraints** | none |
| **Coordinate-components** | $u$, $v$ |
| **Domain of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Generating function or mapping equations** | $\boldsymbol{F}_{\text{E2D}}(u,v) = (x,y)$,<br>where:<br>$x = u$, and<br>$y = v$. |

| Element | Specification |
| --- | --- |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}_{\text{E2D}}^{-1}(x, y) = (u, v)$,<br>where:<br>$u = x$, and<br>$v = y$. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |
| **Notes** | Coordinate-space 2-tuples are identified with position-space 2-tuples. |
| **References** | [EDM] |

### 5.9.24 Lococentric Euclidean 2D CS specification

**Table 5.30 — Lococentric Euclidean 2D CS**

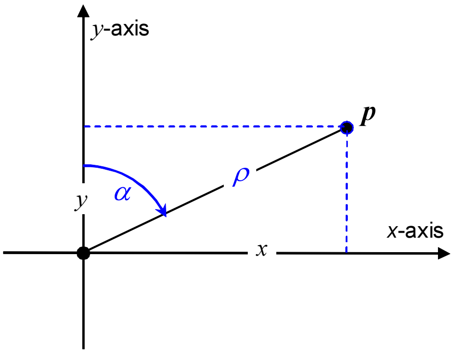
| Element | Specification |
| --- | --- |
| **Description** | Localized Euclidean 2D |
| **CS label** | LOCOCENTRIC_EUCLIDEAN_2D |
| **CS code** | 23 |
| **Function type** | Generating function. |
| **CS descriptor** | 2D linear. |
| **Properties** | Orthonormal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^2$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^2$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |

| Element | Specification |
|---|---|
| **Coordinate-components** | $u$, $v$ |
| **Domain of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(u,v) = \boldsymbol{L}_{\text{2D}} \circ \boldsymbol{F}_{\text{E2D}}(u,v)$,<br>where:<br>$\boldsymbol{L}_{\text{2D}}$ = the 2D localization operator, and<br>$\boldsymbol{F}_{\text{E2D}}$ = the Euclidean 2D CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y) = \boldsymbol{F}_{\text{E2D}}^{-1} \circ \boldsymbol{L}_{\text{2D}}^{-1}(x,y)$,<br>where:<br>$\boldsymbol{L}_{\text{2D}}^{-1}$ = the 2D localization inverse operator, and<br>$\boldsymbol{F}_{\text{E2D}}^{-1}$ = the Euclidean 2D CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |   |
| **Notes** | 1) Euclidean 2D CS is a special case with $\boldsymbol{q} = (0,0)$, $\boldsymbol{r} = (1,0)$, $\boldsymbol{s} = (0,1)$.<br>2) The generating function is the composition of the generating function for Euclidean 2D CS (see Table 5.29) with the 2D localization operator (see 5.7). |
| **References** | [EDM] |

### 5.9.25 Azimuthal CS specification

**Table 5.31 — Azimuthal CS**

| Element | Specification |
|---|---|
| **Description** | Azimuthal coordinate system. |
| **CS label** | AZIMUTHAL |
| **CS code** | 24 |
| **Function type** | Generating function. |
| **CS descriptor** | 2D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | none |
| **Coordinate-components** | $\alpha$ : azimuth in radians, and<br>$\rho$ : radius. |
| **Domain of the generating function or mapping equations** | $\left\{(\alpha,\rho) \text{ in } \mathbf{R}^2 \;\middle|\; 0 \le \alpha < 2\pi, \text{ and } 0 < \rho\right\} \cup \left\{(0,0)\right\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}_{\mathrm{A}}(\alpha,\rho) = (x,y)$,<br>where:<br>$x = \rho\sin(\alpha)$, and<br>$y = \rho\cos(\alpha)$. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y,z) = (\alpha,\rho)$,<br>where:<br>$\alpha = \begin{cases} \alpha' & \text{if } \alpha' \ge 0 \\ 2\pi + \alpha' & \text{if } \alpha' < 0 \end{cases}$,<br>$\alpha' = \operatorname{arctan2}(x,y)$, and<br>$\rho = \sqrt{x^2 + y^2}$. |
| **COM** | n/a |
| **Point distortion** | n/a |

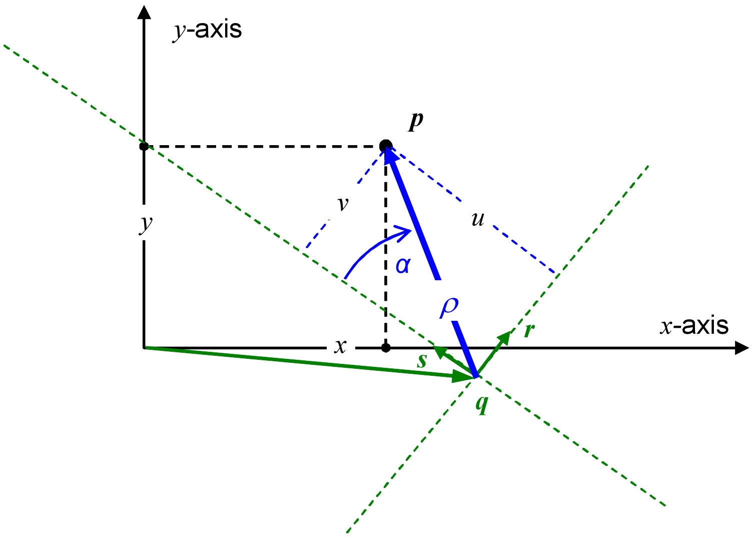
| Element | Specification |
|---|---|
| **Figures** |  |
| **Notes** | The inverse generating function is discontinuous at the CS domain boundary point (0, 0). |
| **References** | [EDM] |

### 5.9.26 Lococentric azimuthal CS specification

**Table 5.32 — Lococentric azimuthal CS**

| Element | Specification |
|---|---|
| **Description** | Localization of azimuthal CS. |
| **CS label** | LOCOCENTRIC_AZIMUTHAL |
| **CS code** | 25 |
| **Function type** | Generating function. |
| **CS descriptor** | 2D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^2$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^2$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $\alpha$: azimuth in radians, and<br>$\rho$: radius or range. |
| **Domain of the generating function or mapping equations** | $\left\{(\alpha,\rho) \text{ in } \mathbf{R}^2 \mid 0 \le \alpha < 2\pi, \text{ and } 0 < \rho\right\} \cup \left\{(0,0)\right\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\alpha,\rho) = \boldsymbol{L}_{\text{2D}} \circ \boldsymbol{F}_{\text{A}}(\alpha,\rho)$,<br>where:<br>$\boldsymbol{L}_{\text{2D}}$ = the 2D localization operator, and<br>$\boldsymbol{F}_{\text{A}}$ = the azimuthal CS generating function. |

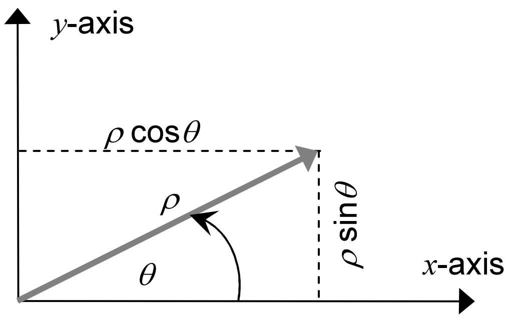
| Element | Specification |
|---|---|
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y) = \boldsymbol{F}_{\mathrm{A}}^{-1} \circ \boldsymbol{L}_{\mathrm{2D}}^{-1}(x,y),$<br>where:<br>$\boldsymbol{L}_{\mathrm{2D}}^{-1}$ = the 2D localization inverse operator, and<br>$\boldsymbol{F}_{\mathrm{A}}^{-1}$ = the azimuthal CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |
| **Notes** | The generating function is the composition of the generating function for azimuthal CS (see Table 5.31) with the 2D localization operator (see 5.7). |
| **References** | [EDM] |

### 5.9.27 Polar CS specification

**Table 5.33 — Polar CS**

| Element | Specification |
|---|---|
| **Description** | Polar coordinate system. |
| **CS label** | POLAR |
| **CS code** | 26 |
| **Function type** | Generating function. |
| **CS descriptor** | 2D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | none |

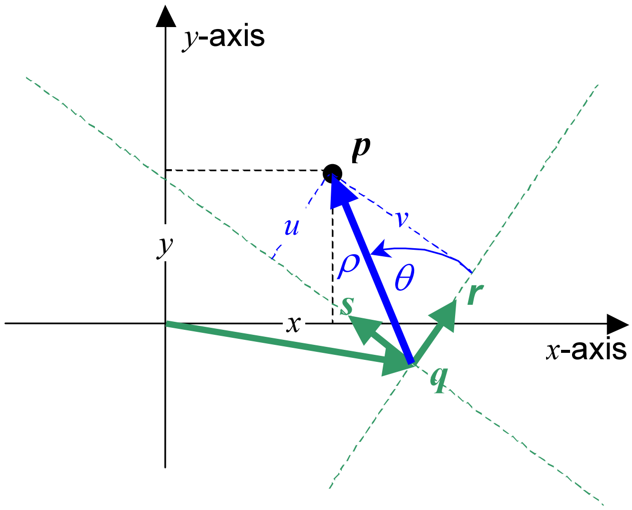
| Element | Specification |
|---|---|
| **Coordinate-components** | $\rho$: radius, and<br>$\theta$: angle in radians. |
| **Domain of the generating function or mapping equations** | $\{(\rho,\theta) \text{ in } \mathbf{R}^2 \mid 0 < \rho \text{ and } 0 \le \theta < 2\pi\} \cup \{(0,0)\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}_{\mathrm{p}}(\rho,\theta) = (x, y)$,<br>where:<br>$x = \rho\cos(\theta)$, and<br>$y = \rho\sin(\theta)$. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}_{\mathrm{p}}^{-1}(x, y) = (\rho,\theta)$,<br>where:<br>$\theta = \begin{cases} \theta' & \text{if } \theta' \ge 0 \\ 2\pi + \theta' & \text{if } \theta' < 0 \end{cases}$<br>$\theta' = \arctan 2(y, x)$, and<br>$\rho = \sqrt{x^2 + y^2}$. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** |  |
| **Notes** | The inverse generating function is discontinuous at the CS domain boundary point (0, 0). |
| **References** | [EDM] |

### 5.9.28 Lococentric polar CS specification

**Table 5.34 — Lococentric polar CS**

| Element | Specification |
|---|---|
| **Description** | Localized polar. |
| **CS label** | LOCOCENTRIC_POLAR |

| Element | Specification |
|---|---|
| **CS code** | 27 |
| **Function type** | Generating function. |
| **CS descriptor** | 2D curvilinear. |
| **Properties** | Orthogonal. |
| **CS parameters and constraints** | Localization parameters:<br>$\boldsymbol{q}$**:** the lococentric origin in $\mathbf{R}^2$, and<br>$\boldsymbol{r}$, $\boldsymbol{s}$**:** axis directions in $\mathbf{R}^2$.<br>Constraints:<br>$\boldsymbol{r}$ and $\boldsymbol{s}$ are orthonormal. |
| **Coordinate-components** | $\rho$: radius, and<br>$\theta$: angle in radians. |
| **Domain of the generating function or mapping equations** | $\left\{(\rho,\theta) \text{ in } \mathbf{R}^2 \mid 0<\rho \text{ and } 0 \le \theta < 2\pi\right\} \cup \left\{(0,0)\right\}$ |
| **Generating function or mapping equations** | $\boldsymbol{F}(\rho,\theta) = \boldsymbol{L}_{\text{2D}} \circ \boldsymbol{F}_{\text{P}}(\rho,\theta)$,<br>where:<br>$\boldsymbol{L}_{\text{2D}}$ = the 2D localization operator, and<br>$\boldsymbol{F}_{\text{P}}$ = the polar CS generating function. |
| **Domain of the inverse of the generating function or mapping equations** | $\mathbf{R}^2$ |
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}^{-1}(x,y) = \boldsymbol{F}_{\text{P}}^{-1} \circ \boldsymbol{L}_{\text{2D}}^{-1}(x,y)$,<br>where:<br>$\boldsymbol{L}_{\text{2D}}^{-1}$ = the 2D localization inverse operator, and<br>$\boldsymbol{F}_{\text{P}}^{-1}$ = the polar CS inverse generating function. |
| **COM** | n/a |
| **Point distortion** | n/a |

| Element | Specification |
|---|---|
| Figures |  |
| Notes | The generating function is the composition of the generating function for the polar CS (see Table 5.33) with the 2D localization operator (see 5.7). |
| References | [EDM] |

#### 5.9.29 Euclidean 1D CS specification

**Table 5.35 — Euclidean 1D CS**

| Element | Specification |
|---|---|
| Description | Euclidean 1D. |
| CS label | EUCLIDEAN_1D |
| CS code | 28 |
| Function type | Generating function. |
| CS descriptor | 1D linear |
| Properties | none |
| CS parameters and constraints | none |
| Coordinate-components | $u$ |
| Domain of the generating function or mapping equations | $\mathbf{R}^1$ |
| Generating function or mapping equations | $F_{\text{E1D}}(u) = (x)$<br>where:<br>$x = u.$ |
| Domain of the inverse of the generating function or mapping equations | $\mathbf{R}^1$ |

| Element | Specification |
|---|---|
| **Inverse of the generating function or mapping equations** | $\boldsymbol{F}_{\mathrm{E1D}}^{-1}(x) = (u)$, where: $u = x$. |
| **COM** | n/a |
| **Point distortion** | n/a |
| **Figures** | none |
| **Notes** | Coordinate-space 1-tuples are identified with position-space 1-tuples. |
| **References** | [EDM] |

http://standards.iso.org/ittf/PubliclyAvailableStandards/